Advanced Information Engineering

#11 December 21 (Mon), 2020

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Frequency Characteristic of System

- One of the purposes of filtering is to pass specific frequency components.
- The filter of a linear shift-invariant system is evaluated by frequency characteristic.

Frequency Characteristic

- Since it is the same equation for discrete spatial Fourier transform, if the signal is even symmetric, the frequency characteristic is real-valued.

$$
\begin{aligned}
H(e^{j\omega_1}, e^{j\omega_2}) &= \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} h(n_1, n_2) e^{-j(\omega_1 n_1 + \omega_2 n_2)} \\
&= A(\omega_1, \omega_2) e^{j\theta(\omega_1, \omega_2)}
\end{aligned}
$$

Derivation of Frequency Characteristic

- For a linear shift-invariant system, input a complex sinusoidal signal, then….

$$y(n_1,n_2) = \sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} h(k_1,k_2)e^{j(\omega_1(n_1-k_1)+\omega_2(n_2-k_2))}$$

$$= \left(\sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty} h(k_1,k_2)e^{-j(\omega_1 k_1+\omega_2 k_2)}\right) e^{j(\omega_1 n_1+\omega_2 n_2)}$$

$$\begin{aligned} y(n_1,n_2) &= H(e^{j\omega_1},e^{j\omega_2})e^{j(\omega_1 n_1+\omega_2 n_2)} \\ &= A(\omega_1,\omega_2)e^{j\theta(\omega_1,\omega_2)}e^{j(\omega_1 n_1+\omega_2 n_2)} \\ &= A(\omega_1,\omega_2)e^{j((\omega_1 n_1+\omega_2 n_2)+\theta(\omega_1,\omega_2))} \end{aligned}$$

Z Transform and Transfer Function

- To represent a linear shift-invariant system compactly, we define a transfer function based on Z transform.

2 次元信号 $g(n_1, n_2)$ に対する z 変換は,

$$G(z_1, z_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} g(n_1, n_2) z_1^{-n_1} z_2^{-n_2}$$

Z Transform

- Transform defined in discrete space（Laplace transform）
- By substituting Z_1, Z_2 with $e^{j\omega_1}$ and $e^{j\omega_2}$を, we will obtain discrete space Fourier transform.

2次元信号 $g(n_1, n_2)$ に対する z 変換は，

$$G(z_1, z_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} g(n_1, n_2) z_1^{-n_1} z_2^{-n_2}$$

Properties of Z Transform

- Linearity, separability for separable signals, convolution, shift of signals
- Similar properties with Fourier transform

性質	信号	変換領域
線形性	$a_1 g_1(n_1, n_2) + a_2 g_2(n_1, n_2)$	$a_1 G_1(z_1, z_2) + a_2 G_2(z_1, z_2)$
可分性	$g_1(n_1) g_2(n_2)$	$G_1(z_1) G_2(z_2)$
たたみ込み	$h(n_1, n_2) * x(n_1, n_2)$	$H(z_1, z_2) X(z_1, z_2)$
シフト	$g(n_1 - k_1, n_2 - k_2)$	$G(z_1, z_2) z_1^{-k_1} z_2^{-k_2}$

Transfer Function

- Convolution

$$y(n_1, n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} h(k_1, k_2) x(n_1 - k_1, n_2 - k_2)$$
$$= h(n_1, n_2) * x(n_1, n_2)$$

- From the properties of Z transform

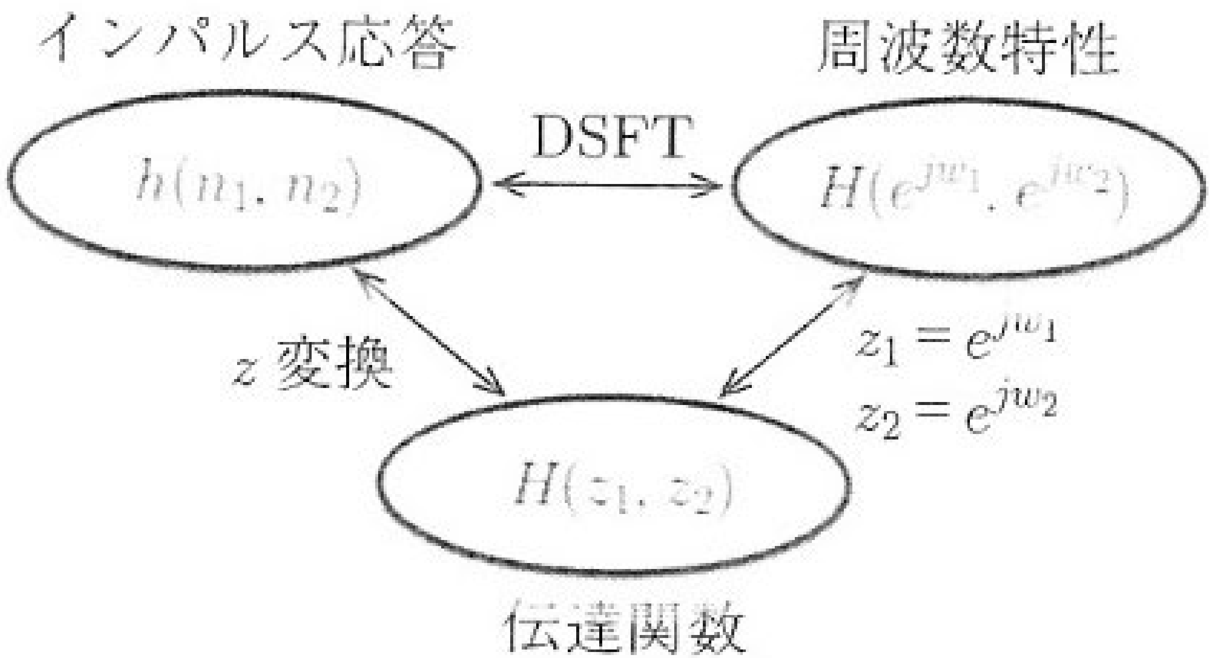
$$Y(z_1, z_2) = H(z_1, z_2) X(z_1, z_2)$$

where $H(z_1,z_2)$ is called a transfer function.

- Ex. Point spread function of optical microscopy and optical transfer function

Transfer Function

- The frequency characteristic is obtained from its transfer function.
- When the impulse response is a separable signal, its filter becomes a separable filter because of the properties of Z transform.



Exercise Example

- Find the transfer function of the following linear shift-invariant system.
- Show that the system is separable.

$$\text{(c)}\ y(n_1,n_2) = \{x(n_1,n_2) + x(n_1-1,n_2) + x(n_1,n_2-1) + x(n_1-1,n_2-1)\}/4$$

Answer

- Input an unit impulse signal to the system and obtain the impulse response.
- Perform Z transform for the impulse response and get the transfer function.

$$H(z_1, z_2) = 1/4 \cdot (1 + z_1^{-1} + z_2^{-1} + z_1^{-1} z_2^{-1}) = 1/4 \cdot (1 + z_1^{-1})(1 + z_2^{-1})$$

Separable Filter

- Most of the image processing are performed based on separable filters and have the following properties.

- Multi-dimensional filter can consist of iterations of 1D filter processing.

- Its computational cost is relatively low.

- There are limits on realizable frequency characteristics.

Separable Filter

- Multi-dimensional filter can consist of iterations of 1D filter processing.
- Its computational cost is relatively low.

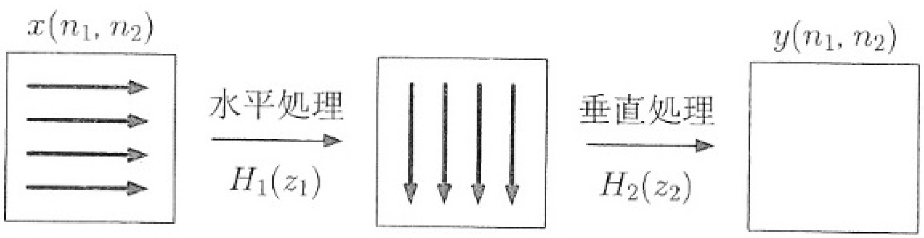
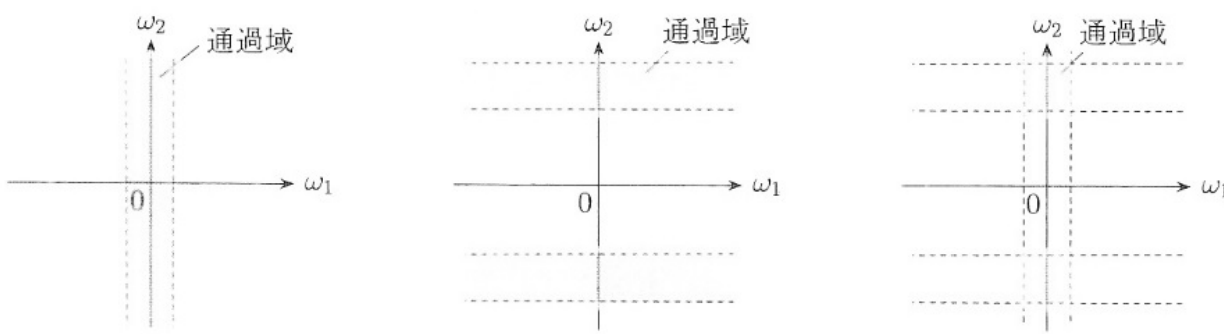


図 3.17　分離型フィルタの実行手順



(a) $H_1(z_1)$, LPF　(b) $H_2(z_2)$, BPF　(c) $H(z_1, z_2) = H_1(z_1)H_2(z_2)$

図 3.19　分離型フィルタの振幅特性

Separable Filter

- There are limits on realizable frequency characteristics.
- Which can be realizable by a separable filter?

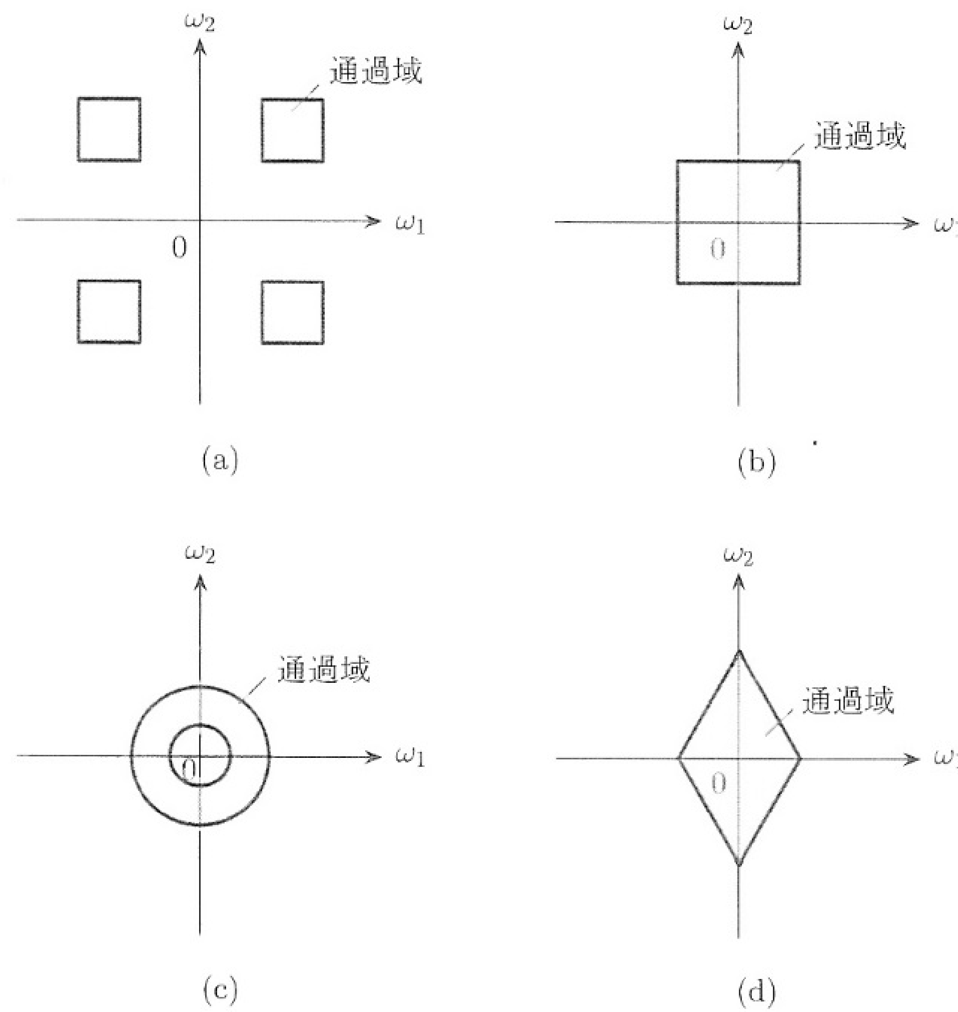


図 **3.20**　例題 3.9

Filter Processing in the Frequency Domain

- Can be perform by FFT.
- Manipulate image spectrum directly.

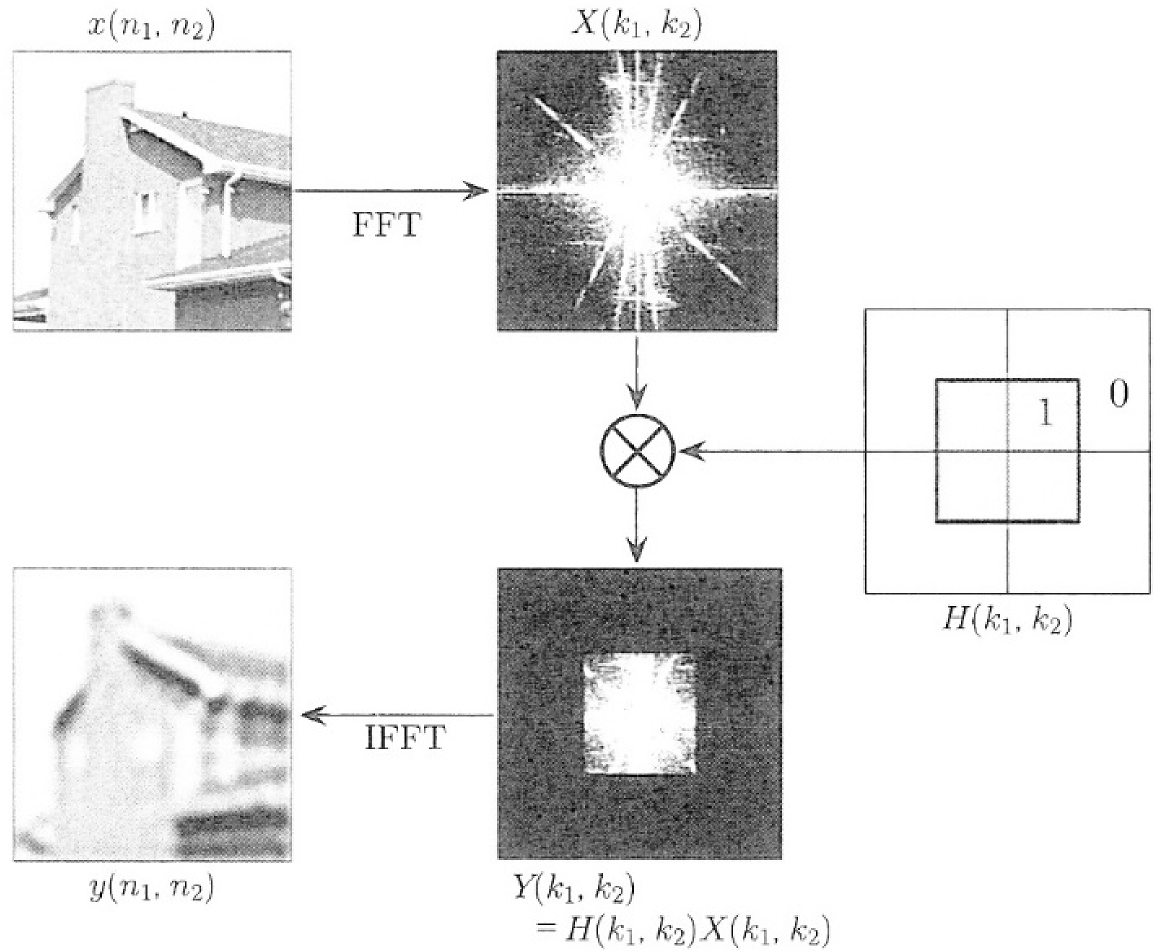


図 3.21　周波数領域でのフィルタ処理手順

Enhancement of Image

- Most of the purposes of image signal processing are to enhance necessary information in the images for individual applications.

Classification of Image Enhancement

- Classified into pixel processing and local processing.
- Filter processing we have already studied is classified as local processing.

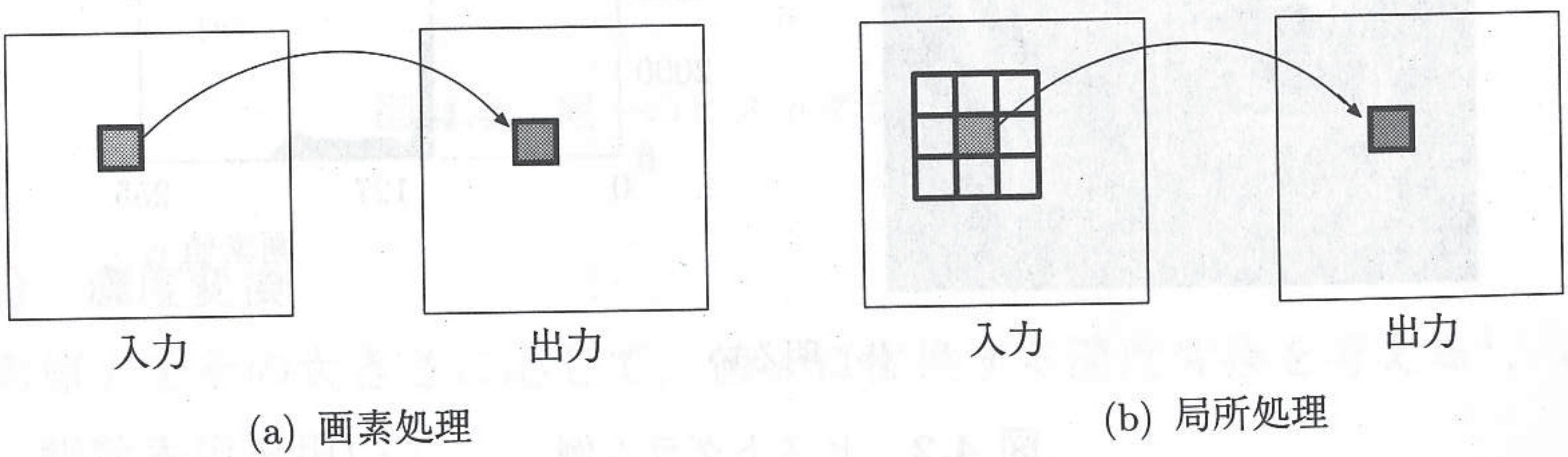
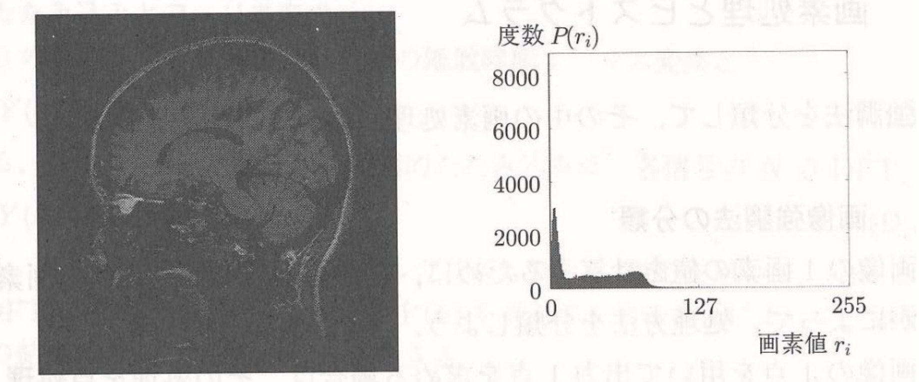


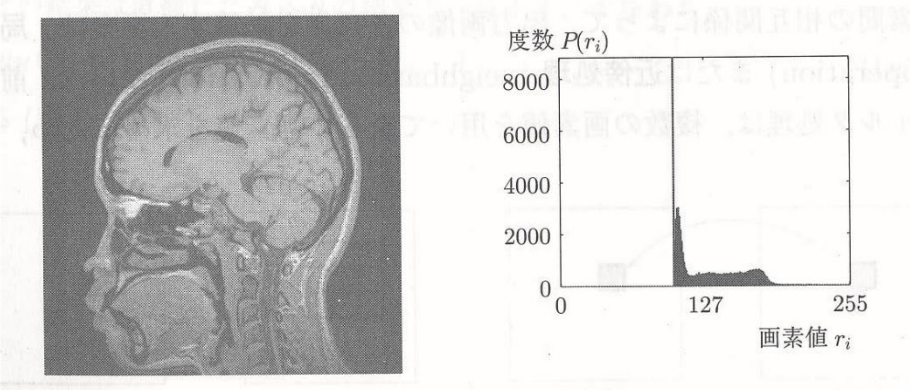
図 4.1 局所処理と画素処理

Histogram

- Correct histogram to image enhancement.
- The following images are CT or MRI with 512×512 pixels and 8 bits/pixel. Only change lightness and contrast is low.



(a) 暗い



(b) 明るい

Histogram

- The left image is of high contrast and the graph is histogram.
- From the right image we find that the histogram doesn't have spatial information although it has important information.

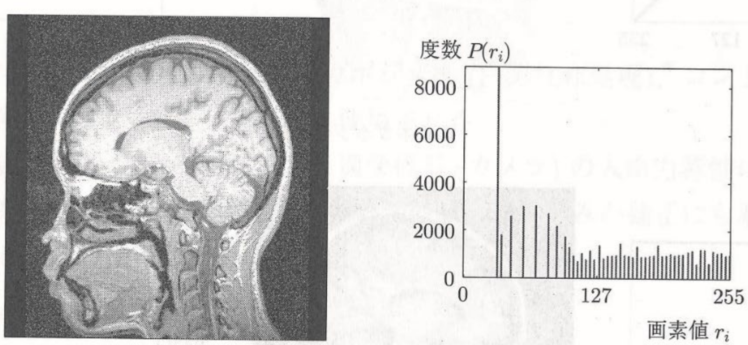


図 4.3　高コントラスト画像

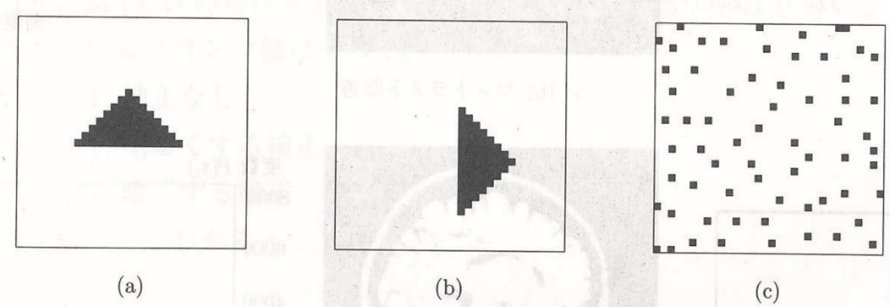


図 4.4　同一のヒストグラムを持つ画像例

Density conversion

- Convert intensity r to s according to r.

$$s = T[r]$$

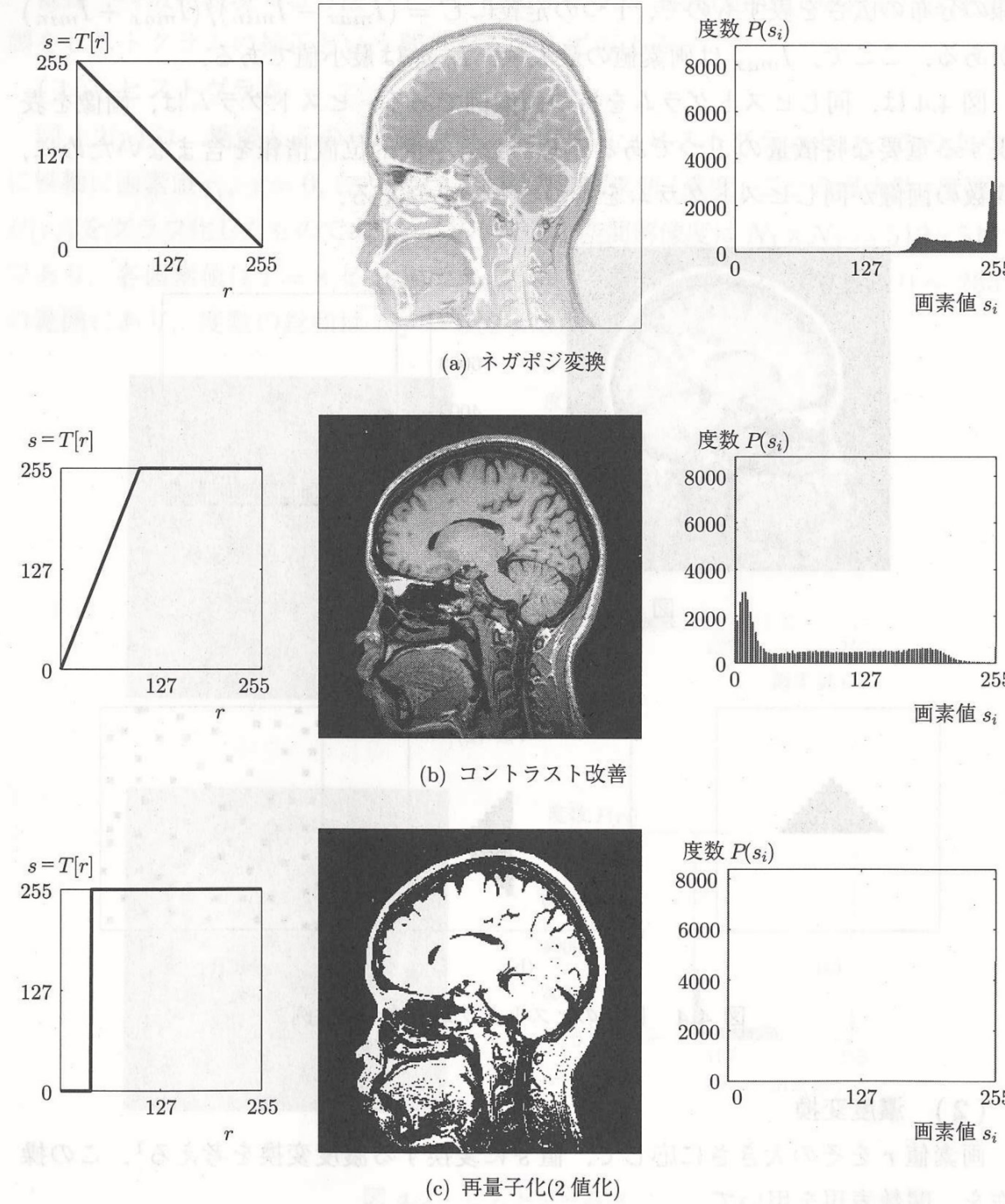


図 4.5　濃度変換関数と変換例

γ Correction

- Input/output characteristic of imaging and displaying devices is generally non-linear and the output signal is distorted.
- In order to correct the distortion, γ correction is used which is one of density conversion.
- How is the intensity converted?

$$s = T[r] = r^{\gamma}$$

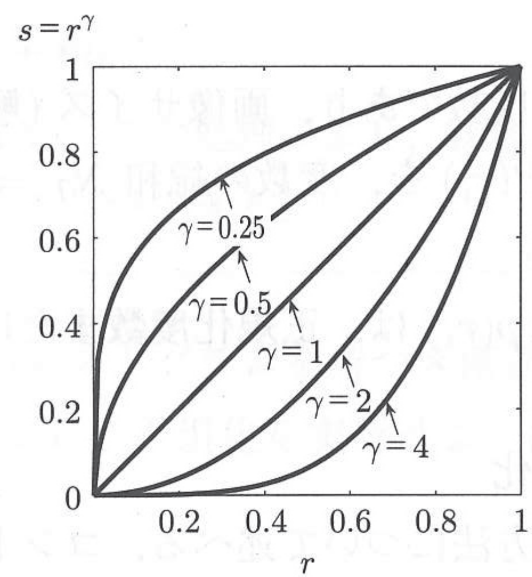


図 **4.6** ガンマ (γ) 補正

Normalization of Intensity

- The values of the histogram depends on the image resolution, so it is usually normalized.

そこで，画素値 r_i の度数 $P(r_i)$ を，度数の総和 $N_T = N_1 \times N_2$

$$p(r_i) = P(r_i)/N_T$$

Histogram Flattening

- Histogram flattening can perform image enhancement (high contrast).

階調数 $L = 2^l$ の画像において，画素値 r_i, $(r_i < r_{i+1})$, $i = 0, 1, \cdots, L-1$ を s_i に変換する濃度変換関数は

$$s_i = T[r_i] = (L-1)\sum_{m=0}^{i} P(r_m)/N_T$$

$$= (L-1)\sum_{m=0}^{i} p(r_m) \tag{4.4}$$

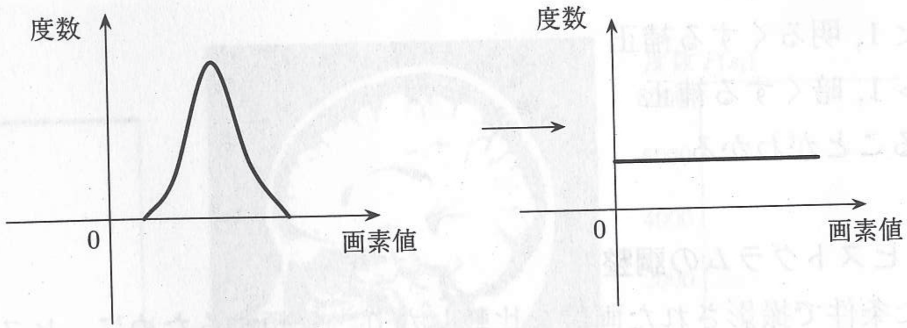


図 4.7　ヒストグラムの平坦化処理

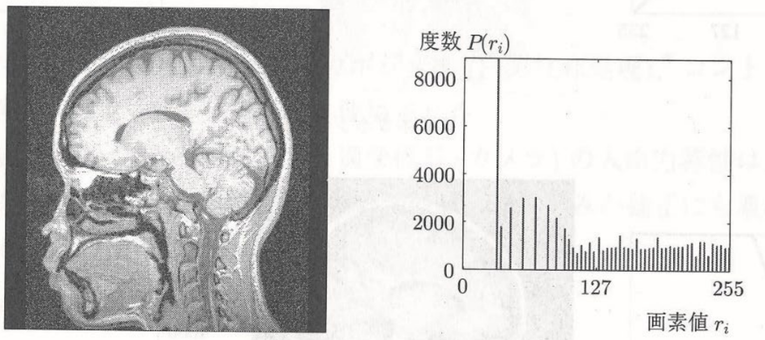
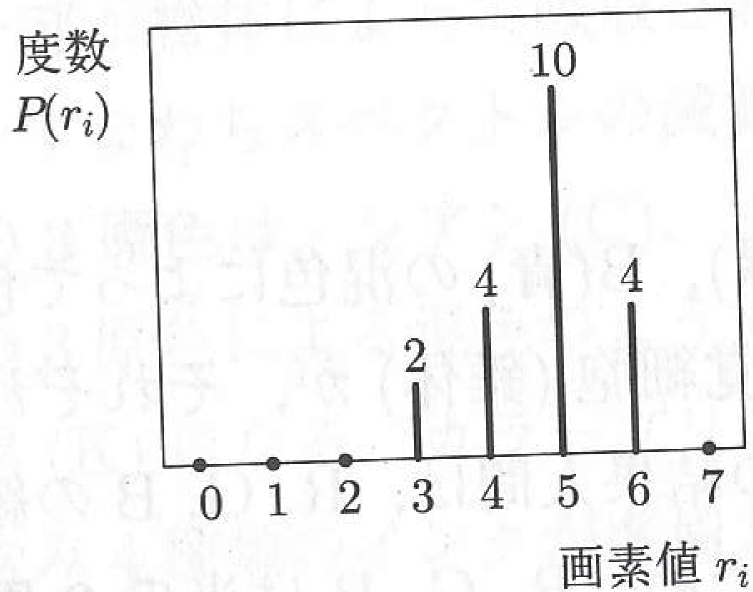


図 4.3　高コントラスト画像

Exercise Example

【例題 4.1】 図 4.8(a) のヒストグラムを持つ 8 階調の画像を考える．(a) 式 (4.4) に基づくヒストグラムの平坦化を実行せよ．(b) 平均値 m，標準偏差 σ を求めよ．



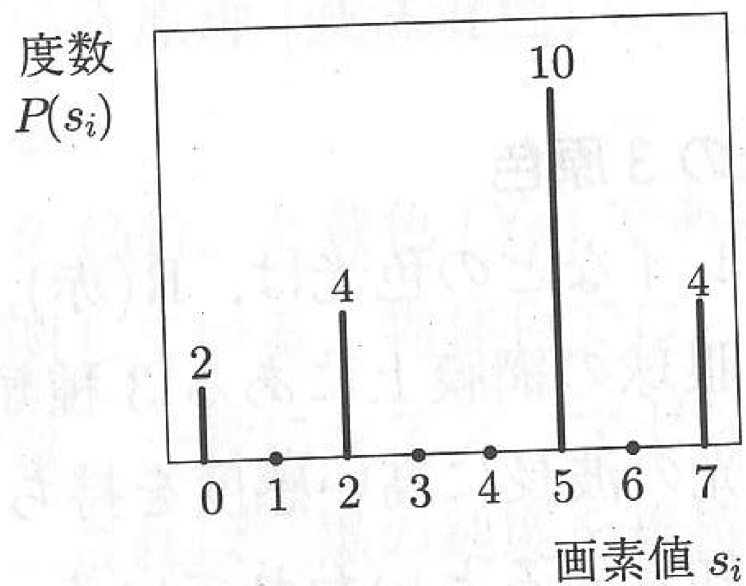
(a) ヒストグラム例

図 4.8

回答(a)

表 4.1　例題 4.1 の計算結果

i	0	1	2	3	4	5	6	7
r_i	0	1	2	3	4	5	6	7
$P(r_i)$	0	0	0	2	4	10	4	0
$p(r_i)$	0	0	0	2/20	4/20	10/20	4/20	0
s_i	0	0	0	14/20	42/20	112/20	140/20	140/20
s_i(整数化)	0	0	0	0	2	5	7	7



(b) 平担化処理結果

回答(b)

表 4.1　例題 4.1 の計算結果

i	0	1	2	3	4	5	6	7
r_i	0	1	2	3	4	5	6	7
$P(r_i)$	0	0	0	2	4	10	4	0
$p(r_i)$	0	0	0	2/20	4/20	10/20	4/20	0
s_i	0	0	0	14/20	42/20	112/20	140/20	140/20
s_i(整数化)	0	0	0	0	2	5	7	7

$$m = \frac{1}{N_T}\sum_{i=0}^{7} r_i \cdot P(r_i) = 4.8$$

$$\sigma = \left(\frac{1}{N_T}\sum_{i=0}^{7}(r_i - m)^2 \cdot P(r_i)\right)^{1/2} \simeq 0.8718$$

Color Image Signal

- The display uses additive color mixing of RGB colors to represent colors.
- If two colors are mixed and become white, the these colors are called complementary colors.
- Complementary color of red ?
- Complementary color of white ?

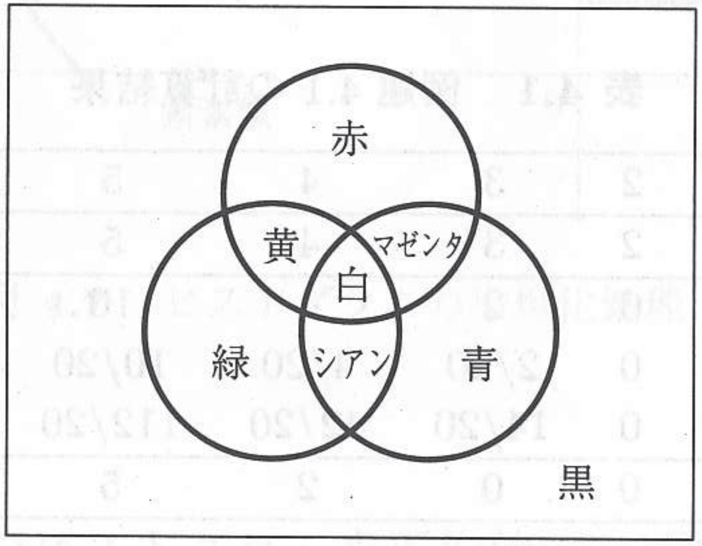


図 4.9　加法混色

The 3 Primary Colors of the Object

- Object Color(CMY) : Color of surface and reflection light when the object is lit by white light.
- Some spectrum of white light is absorbed by the object (subtractive color mixing) .
- Color printer uses CMY inks. Is black necessary ?

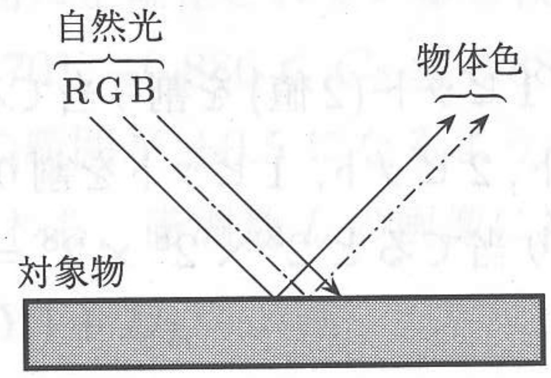


図 4.10　物体色の概念

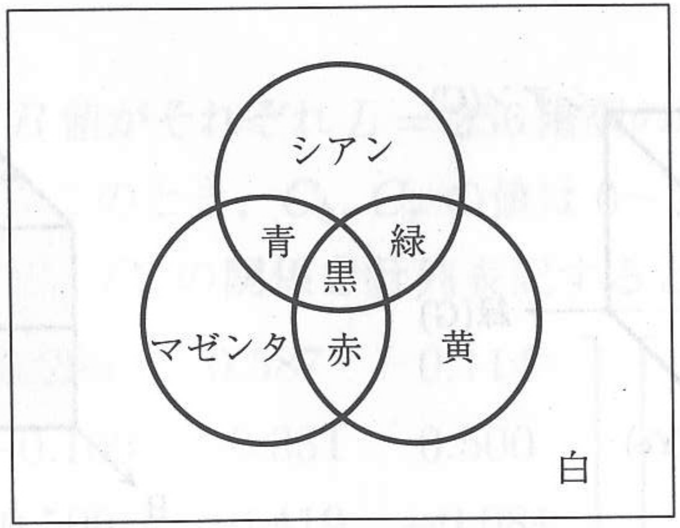


図 4.11　減法混色

RGB Color Space

- Color image signal usually consists of three signal values.
- RGB color space is a 3D space using the orthogonal coordinate system of RGB based on additive color mixing.
- How many colors can be represented when 1 bit is assigned to each of RGB ?
- How many colors can be represented when 8 bits are assigned to each of RGB ?

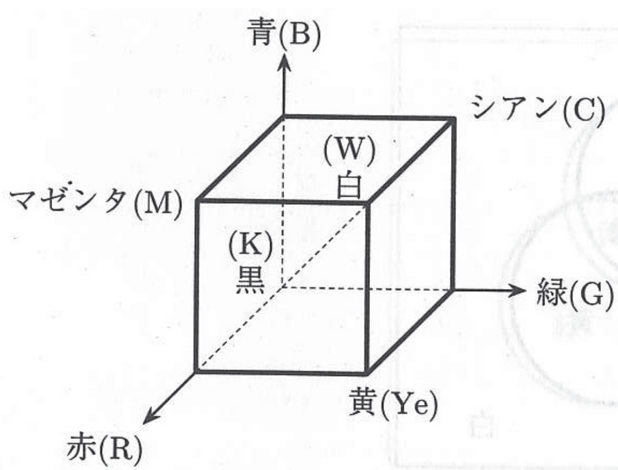


図 4.12　RGB 色空間

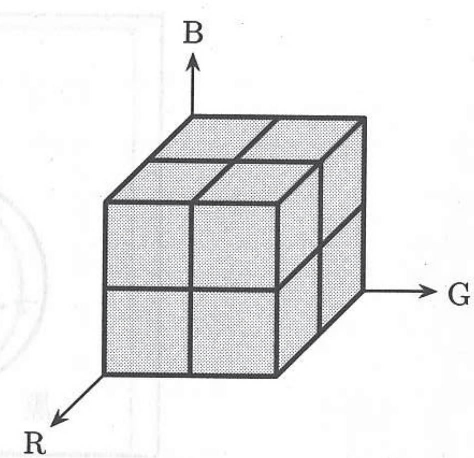


図 4.13　RGB 色空間の分割 (8 色)

Luminance and Color Difference Space

- The human visual characterristics are different for luminance and color, color signal is subdivided into luminance and color information.
- YC_bC_r is used.
- Luminance $Y=0.299R+0.587G+0.114B$

(This is used for monochromatic signal converted from color image signal.)

- C_b :color difference B, C_r:color difference
- For a image with tone L, offset D_L is to be L/2,

$$C_b=(0.5/0.886)(B-Y)+D_L$$

$$C_r=(0.5/0.701)(R-Y)+D_L$$

Luminance and Color Difference Space

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.500 \\ 0.500 & -0.419 & -0.081 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 0 \\ D_L \\ D_L \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.402 \\ 1 & -0.344 & -0.714 \\ 1 & 1.722 & 0 \end{bmatrix} \begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} + \begin{bmatrix} 0 \\ D_L \\ D_L \end{bmatrix}$$

Color Space

- RGB color space is suitable for display and imaging devices.
- YC_bC_r color space is suitable for image transfer and compression.
- The human vision’s sensitivity in the high frequency domain is worse for color difference than luminance and there are compression color format reducing color difference signal.
- There are other various color space adjusted to human vision and devices.

Example Exercise

- Find the formula to convert RGB values to CMY values of a pixel. We assume that RGB values are normalized as 0～1.

Answer

Ye, C, M は，R, G, B の補色である．ゆえに，それらの相互変換は，

$$\begin{bmatrix} C \\ M \\ Ye \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

ここで，値 1 は白色を意味する．