Advanced Information Engineering

#2 October 12 (Mon), 2020 Kenjiro T. Miura

Image Signals

 Image information, in many cases, is transformed to electrical signals or numerical values and is treated as image signals.

Generation of Image Signals

- Light is reflected on subjects.
- Reflected light is refracted and imaged.
- Image signals are recorded on photoreceptor as film, or transformed to electrical signals by imaging devices.



Dimension of Signals

- Signals, whose typical example is sound, is a function of time g_a(t) of one dimensional signal.
- Still image is two dimensional signals and has various values at spacial position (x,y).



Digital Image

- Images taken by a digital camera are digital images and they are recorded as digital signals.
- Image processing processes digital signals by computers.

Analog Image Signals

- Intensities of film photo are analog signals and are given by two dimensional signal g_a(x,y).
- Variables x and y are continuous(real numbers) and pixel which is a discrete space point is not defined.
- Signal values g_a(x,y) are continuous (real values) and are not handled by computers because continuous values need infinite number of bits.

Digital Image Signals

- Image signals whose signals themselves g(x,y) and variables x, y are discrete.
- In order to convert analog signals to digital signals, it is necessary to discretize variables (sampling) and signals (quantization).

Sampling

- Process to discretize variables x and y.
- Let T_{s1} , T_{s2} be sampling intervals in x and y directions, respectively. The signal is given by

$$g(n_1, n_2) = g_a(x, y)|_{x=n_1 T_{s_1}, y=n_2 T_{s_2}}$$
$$= g_a(n_1 T_{s_1}, n_2 T_{s_2})$$

- n₁, n₂ are integers and reciprocals of sampling intervals Ts1, Ts2 are horizontal and vertical sampling frequencies.
- A sample point in space is a pixel and its signal value is a pixel value.

Rectangular Sampling

- It is commonly used because of simpleness of output devices and processing.
- The top-left corner is the image origin (0,0) and n₁ and n₂ are variables in the horizontal and vertical directions.



Spatial Resolution of Image

- For a digital image, let N₁, and N₂ be pixel numbers in the horizontal and vertical axes. Its space resolution is given by N₁×N₂.
- Higher space resolution, higher-definition
- VGA, HD, 4K ?





(b) 128×128

(c) 64×64

図 1.4 解像度の異なる画像例

Digitization of Video Image Signal g_a(x,y,t)

 Discretize time variable tを by frame interval T₅.

 $g(n_1, n_2, n) = g_a(n_1 T_{s_1}, n_2 T_{s_2}, n T_s)$

- n is a variable for time elapse and is an integer.
- For example if $T_s = 1/30[sec]$, the frame rate, which corresponds to time resolution is 30fps.
- Fs, the reciprocal of Ts Fs is called time sampling frequency.

Sample Exercise

• Let's think about two one-dimensional time signals of frequency F=1[Hz] and F'=5[Hz], g_a(t)=cos(2π t) and g_a(t)=cos(10π t). Please illustrate discrete signals obtained by sampling these signals with sampling interval T_s=1/F_s=1/4[sec].

Answers

- Their sampled values are the same.
- Sampling time t=nTs, and g(n)=cos(π n/2)=cos(2π + π n/2)=g'(n)
- Generally when F'=F+kF_s (k is an integer), both of the sampled values become identical.



図 1.5 例題 1.2



Process to transform multi-dimensional signals to one dimensional signals.



- TV broadcasting uses interlace scan and movie files do progressive scan.
- Prioritize time resolution or space resolution?

Digital Image Signal (review)

- Image signal g(x, y) and variables x, y are discrete.
- To convert analogue signal to digital signal, discretization of variables (sampling) and that of image signal (quantization) are necessary.

量子化(quantization)

- Because analog signal are real numbers and infinite, for quantization, for example with 8 bits it must be replaced with values $L=2^8=256$.
- Quantization means that values of some number are replaced with other values of smaller number.



量子化(quantization)

- L is called quantization level or quantization step number.
- The difference between maximum and minimum values g_{max} g_{min} is called dynamic range.



量子化(quantization)

- By quantization step number L and dynamic range g_{max} - g_{min}, quantization step Δ=(g_{max} - g_{min})/L If Δ dose not depend on the index, it is called uniform quantization.
- In case where a sample number is a neighborhood of a representative value, it is regarded as quantization value.



量子化誤差(quantization error)

- The difference between a sample value and its quantization value is called quantization error or quantization noise.
- The range of quantization error e is given with quantization step Δ by



量子化誤差(quantization error)

• It is possible to make quantization error smaller by smaller quantization step, but more bits are needed.



Encoding

 Process where signal values quantized with quantization level L are assigned to integer index n: (i=1,2,...,L) which are one to one correspondent with the signal values.



符号化(encoding)

- In case where L=5, since 5≤2³, 3 bits are used for encoding.
- Encoding with 3 bits is given as binary numbers by 000,001,010,011,100.



Digital Image

 The digital signal is an analog signal modified by sampling, quantization and encoding and the image represented by a digital signal is called digital image.

Tone of Image

- For the digital image, quantization level number is called tone.
- What is the tone of 16-bit image ?



(a) 2 階調(1 ビット)





(b) 4 階調(2 ビット)

(c) 256 階調(8 ビット)

図 1.9 階調の異なる画像例

画像の階調(tone)

• The pixel value of a digital image is generally a positive integer.



Bit-rate

- Transmission velocity of data transmission
- Its unit is bps.
- The bit-rate B_r is given by

 $B_r = N_1 \times N_2 \times F_s \times I$

for a gray-scale video with spatial resolution $N_1 \times N_2$, frame-rate F_s , tone $L=2^1$.

• For color video, its bit-rate is 3 times larger.

Exercise #1

 Assume that we use uniform quantization and a sample value g(n)=7. Quantize the value with quantization step Δ=5 and Δ=2,respectively. Note that we are supposed to use rounding (四捨五入). Calculate quantization value and quantization value.

Answer

- When quantization step $\Delta = 5$, quantization value = 5, quantization error = 2.
- When quantization step $\Delta = 2$, quantization value = 8, quantization error -1.

Exercise #2

 We would like to quantize a signal whose dynamic range g_{max} - g_{min} =10 and make quantization error be less than or equal to ¹/₄. How many bits are necessary to encode quantization values?

Answer

- The range of quantization error $-\Delta/2 \leq e < \Delta/2$
- Hence quantization step Δ should be $\frac{1}{2}$.
- Then quantization step number L= 10/(1/2)=20.
- Since $L=20 \le 2^5$, 5 bits are necessary.

Exercise #3

 Calculate bit-rate B_r for color video with its special resolution 1000×1000, frame-rate 30 fps, 8-bit tone. If necessary, please use K (kilo), M (mega), G (giga).

Answer

• Br=1000×1000×30×8×3=720[Mbps]