Advanced Information Engineering

#3 October 19 (Mon), 2020

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Digital Image

- The digital signal is an analog signal modified by sampling, quantization and encoding and the image represented by a digital signal is called digital image.

Sampling

- Process to discretize variables x and y.
- Let T_{s1}, T_{s2} be sampling intervals in x and y directions, respectively. The signal is given by

$$g(n_1, n_2) = g_a(x, y)|_{x=n_1 T_{s_1}, y=n_2 T_{s_2}}$$
$$= g_a(n_1 T_{s_1}, n_2 T_{s_2})$$

- n_1, n_2 are integers and reciprocals of sampling intervals Ts1, Ts2 are horizontal and vertical sampling frequencies.
- A sample point in space is a pixel and its signal value is a pixel value.

量子化(quantization)

- Because analog signal are real numbers and infinite, for quantization, for example with 8 bits it must be replaced with values $L=2^8=256$.
- Quantization means that values of some number are replaced with other values of smaller number.

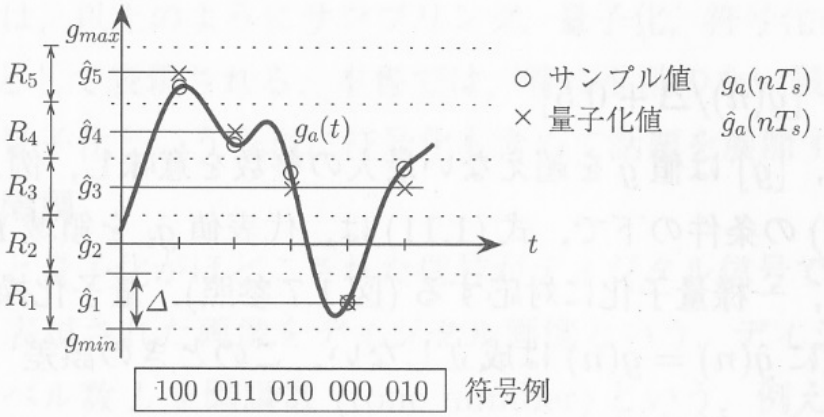

図 1.7　信号の量子化 ($L = 5$)

Encoding

- Process where signal values quantized with quantization level L are assigned to integer index n_i (i=1,2,・・・,L) which are one to one correspondent with the signal values.

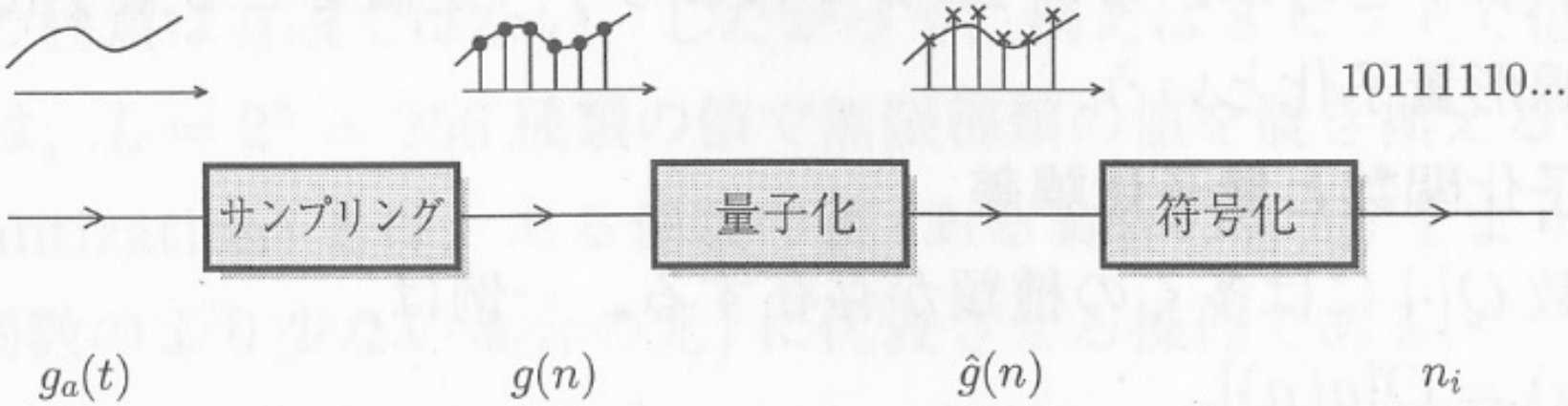

図 1.8　サンプル値の符号化

Tone of Image

- For the digital image, quantization level number is called tone.
- What is the tone of 16-bit image ?

(a) 2 階調(1 ビット)

(b) 4 階調(2 ビット)

(c) 256 階調(8 ビット)

図 1.9　階調の異なる画像例

画像の階調(tone)

- The pixel value of a digital image is generally a positive integer.

図 1.10　8 ビットの画素値表現

Bit-rate

- Transmission velocity of data transmission
- Its unit is bps.
- The bit-rate B_r is given by

$$B_r = N_1 \times N_2 \times F_s \times l$$

for a gray-scale video with spatial resolution $N_1 \times N_2$, frame-rate F_s, tone $L = 2^l$.

- For color video, its bit-rate is 3 times larger.

Exercise #1

- Assume that we use uniform quantization and a sample value g(n)=7. Quantize the value with quantization step Δ=5 and Δ=2,respectively. Note that we are supposed to use rounding (四捨五入). Calculate quantization value and quantization value.

Answer

- When quantization step $\Delta=5$, quantization value = 5, quantization error = 2.
- When quantization step $\Delta=2$, quantization value = 8, quantization error -1.

Exercise #2

- We would like to quantize a signal whose dynamic range $g_{max} - g_{min} = 10$ and make quantization error be less than or equal to ¼. How many bits are necessary to encode quantization values?

Answer

- The range of quantization error

$$-\Delta/2 \leqq e < \Delta/2$$

- Hence quantization step Δ should be ½.
- Then quantization step number L= 10/(1/2)=20.
- Since L=20≦2^5, 5 bits are necessary.

Exercise #3

- Calculate bit-rate B_r for color video with its special resolution 1000×1000, frame-rate 30 fps, 8-bit tone. If necessary, please use K (kilo), M (mega), G (giga).

Answer

- $B_r = 1000 \times 1000 \times 30 \times 8 \times 3 = 720$[Mbps]

Assignment #1
Exercise 1

- Calculate bit-rate of a color (3 channels) video signal with tone 256, spatial resolution 4000 X 3000, 60[fps].

Assignment #1
Exercise 1 answer

- $4000 \times 3000 \times 8 \times 3 \times 60 = 17280000000$
  $= 17.28[\text{Gbit}]$

Assignment #1
Exercise 2

Let's think about a black and white video signal with spatial resolution 1000 X 1000 which has 30 frames per second and whose pixels are represented by 10 bits.

a. Specify spatial resolution and frame rate [fps].
b. Calculate bit-rate [bps].
c. Calculate compression ratio = (compressed file size/original size) in order to transmit at bit-rate 1M[bps].

Exercise 2 answer

a. Bit-rate $1000 \times 1000 \times 10 \times 30 = 300$[Mbps]
b. $1/300 = 0.00333$

Assignment #1
Exercise 3

We quantize signals whose dynamic range is equal to 1 and pixels are represented by 10 bits.

Calculate the maximum value of quantization error.

Exercise 3 answer

- $\Delta = 1/2^{10}$, Maximum quantization error $= \Delta/2 = 1/2048$

Assignment #1
Exercise 4

When we sample signals with frequencies F and $F' = F + k F_s$ where k is integer and F_s is a sampling frequency, prove that their sampling values are identical.

Exercise 4 answer

- By sampling $g_a(t)=\cos(2\pi Ft)$ at $t=n/F_s$
  $g_a(n/F_s)=\cos(2\pi F_n/F_s)$

- By sampling $g_a'(t)=\cos(2\pi(F')t)=\cos(2\pi(F+kF_s)t)$ at $t=n/F_s$

  $g_a'(n/F_s)=\cos(2\pi(F+kF_s)(n/F_s))$

  $=\cos(2\pi F_n/F_s+2\pi kn)$

  $=\cos(2\pi F_n/F_s)$