Advanced Information Engineering

#4 October 26 (Mon), 2020

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Frequency of Image Signal

- Temporal frequency[Hz]or[cycle/sec], cycle number per unit time
- Spatial frequency [cycle/m], cycle number per unit length
- When the pixels are aligned with a specific rule, the reciprocal of their iteration period is called frequency of the image signal.
- Usually the frequency is determined by using 1 dimensional sinusoidal wave of special parameter.
- It is possible to use a cycle around a pixel [cycle/pixel] as unit, but note that it can happen the size of pixels are different.

Space Domain Representation of Signal

- Signal representation by variables (x, y) corresponds to space domain representation.

$$g_a(x,y) = \cos(2\pi F_1 x + 2\pi F_2 y)$$

- What is F_1 ?
- What is F_2 ?
- What is $\Omega_1 = 2\pi F_1$?
- What is $\Omega_2 = 2\pi F_2$?

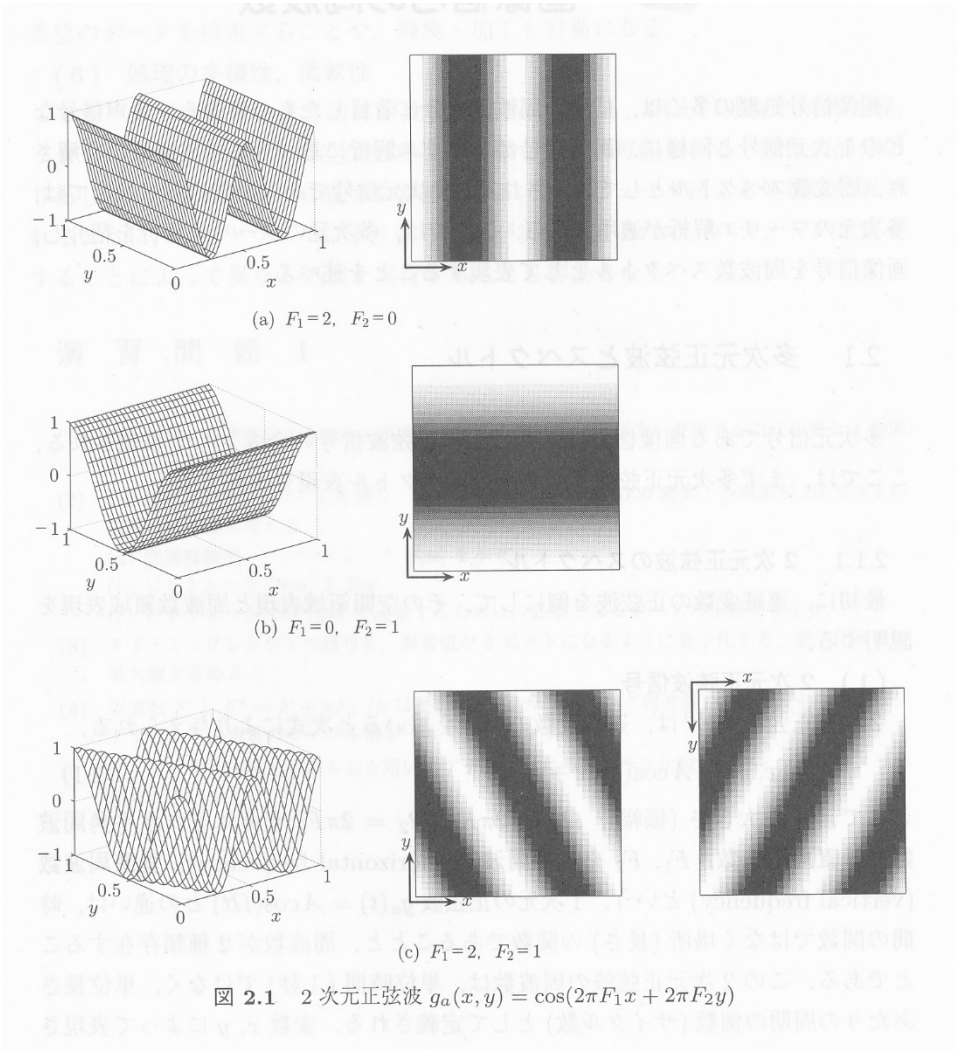


図 2.1　2 次元正弦波 $g_a(x,y) = \cos(2\pi F_1 x + 2\pi F_2 y)$

Spectral Representation of Signal

- Signal representation by frequencies corresponds to frequency domain representation.
- Decompose signals into sinusoidal waves and deal with each frequency component.
- Treatment based on complex sinusoidal signals

$$g_a(x,y) = Ae^{j(\Omega_1 x + \Omega_2 y)}, \quad j = \sqrt{-1}$$

By Euler's formula, g_a is given by

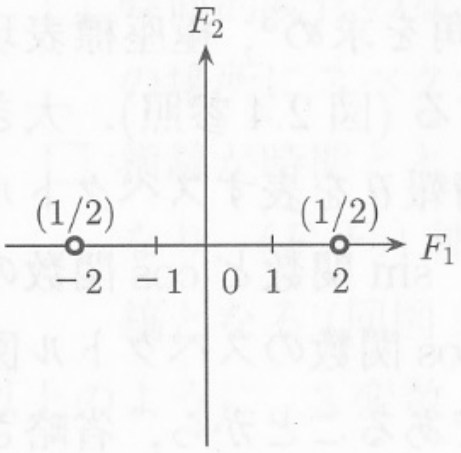
$$g_a(x,y) = \cos(\ \Omega_1\ x + \ \Omega_2\ y)$$

$$g_a(x,y) = (A/2)e^{j(\Omega_1 x + \Omega_2 y)} + (A/2)e^{-j(\Omega_1 x + \Omega_2 y)}$$

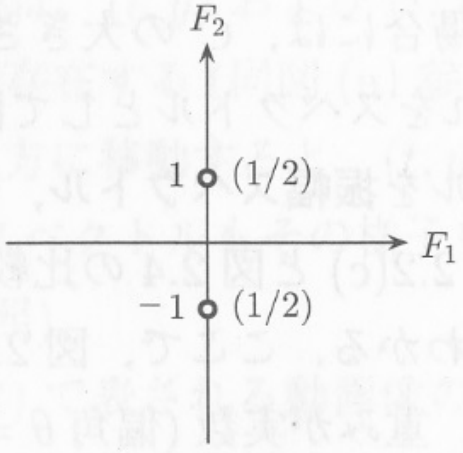
Spectral Representation of Signal

$$g_a(x,y) = (A/2)e^{j(\Omega_1 x + \Omega_2 y)} + (A/2)e^{-j(\Omega_1 x + \Omega_2 y)}$$

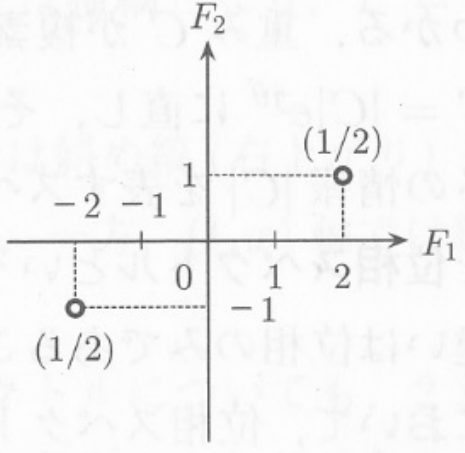
- The signal is represented by the sum of two complex sinusoidal waves with their weight A/2.
- Why is the spectral graph given by the figures below ?
- What is ○ in the spectral graphs?



(a) $F_1 = 2,\ F_2 = 0$



(b) $F_1 = 0,\ F_2 = 1$



(c) $F_1 = 2,\ F_2 = 1$

図 **2.2**　$g_a(x,y) = \cos(2\pi F_1 x + 2\pi F_2 y)$ のスペクトル表現, (·) 内はスペクトル値を示す

Relationship between signal and spectrum

- What is the origin of frequency domain ?
- What will happen if the circle become farther away form the origin?
- What will happen about the direction of modulation ?

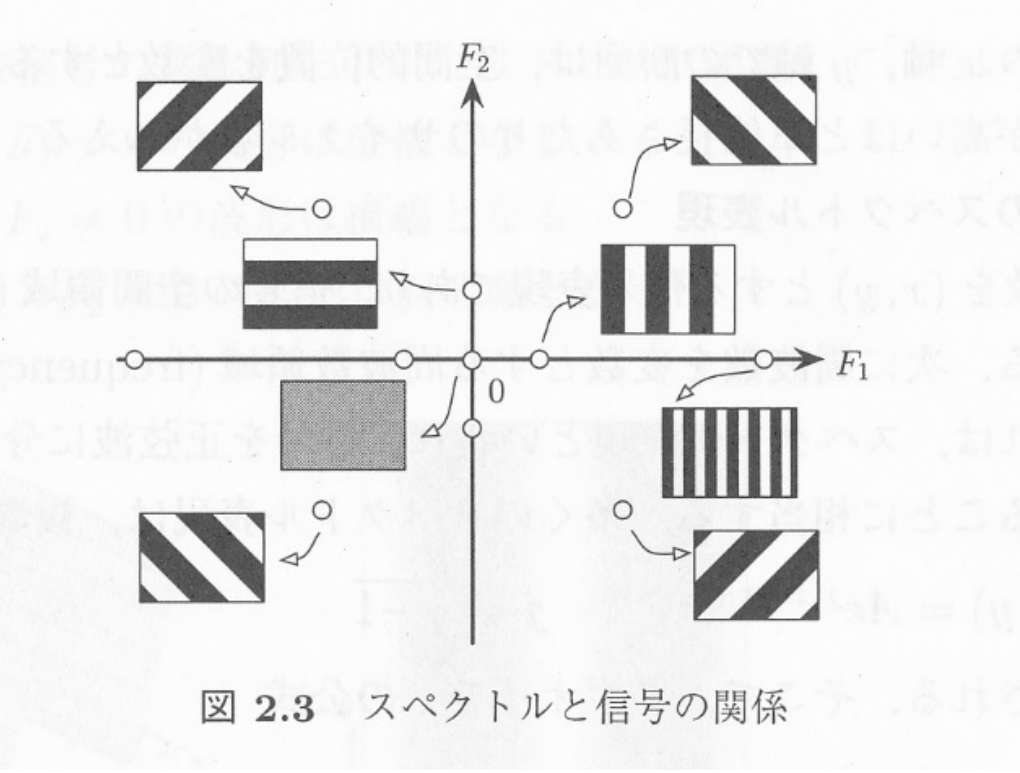


図 2.3　スペクトルと信号の関係

Exercise Example

$$g_a(x,y) = Ae^{j(\Omega_1 x + \Omega_2 y)}, \quad j = \sqrt{-1}$$

How is the above expression rewritten ?

$$g_a(x,y) = \sin\left(\Omega_1 x + \Omega_2 y\right)$$

Answer

$$g_a(x,y) = Ae^{j(\Omega_1 x + \Omega_2 y)}, \quad j = \sqrt{-1}$$

How is the above expression rewritten ?

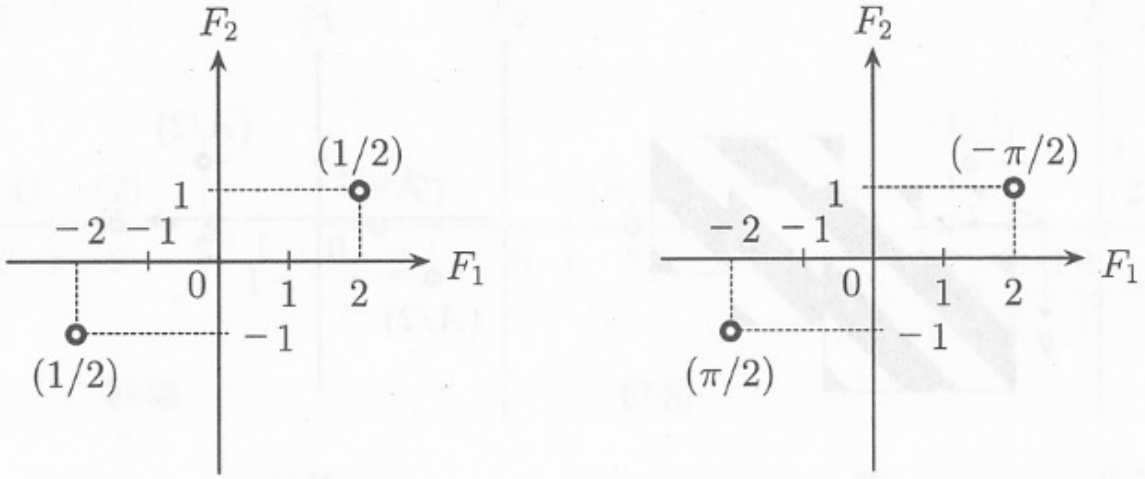
$$g_a(x,y) = \sin\left(\ \Omega_1\ x + \ \Omega_2\ y\right)$$

$$g_a(x,y) = (A/2j)e^{j(\Omega_1 x + \Omega_2 y)} + (-A/2j)e^{-j(\Omega_1 x + \Omega_2 y)}$$
$$= (A/2 \cdot e^{-j\pi/2})e^{j(\Omega_1 x + \Omega_2 y)} + (A/2 \cdot e^{j\pi/2})e^{-j(\Omega_1 x + \Omega_2 y)}$$

Amplitude and Phase Spectra

- When the weight is a complex number, calculate amplitude and phase angle and depict them as amplitude spectrum and phase spectrum.

$$g_a(x,y) = (A/2j)e^{j(\Omega_1 x+\Omega_2 y)} + (-A/2j)e^{-j(\Omega_1 x+\Omega_2 y)}$$
$$= (A/2\cdot e^{-j\pi/2})e^{j(\Omega_1 x+\Omega_2 y)} + (A/2\cdot e^{j\pi/2})e^{-j(\Omega_1 x+\Omega_2 y)}$$



(a) 振幅スペクトル　　(b) 位相スペクトル

図 **2.4**　$g_a(x,y) = \sin(2\pi F_1 x + 2\pi F_2 y), (F_1 = 2, F_2 = 1)$

Spectrum of Video Signals

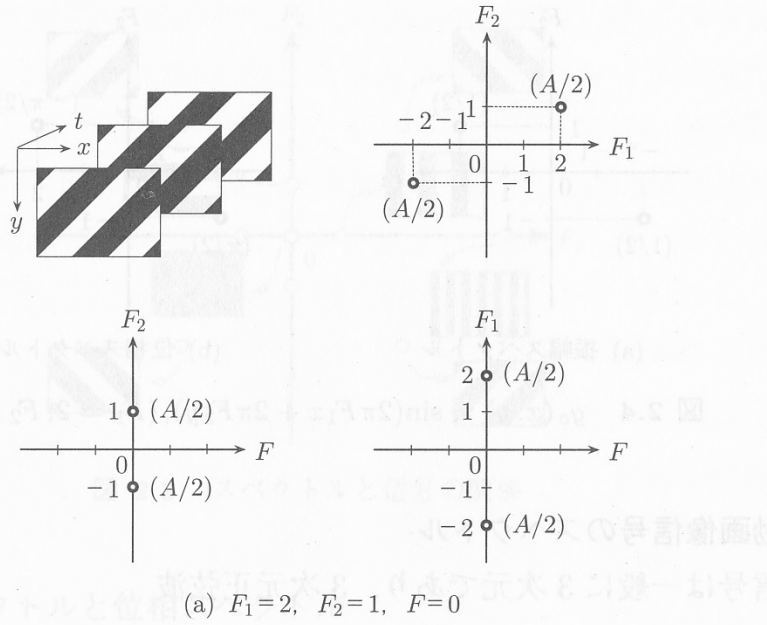
- Video signal is 3-dimensional and given by

$$g_a(x, y, t) = A\cos(\Omega_1 x + \Omega_2 y + \Omega t)$$

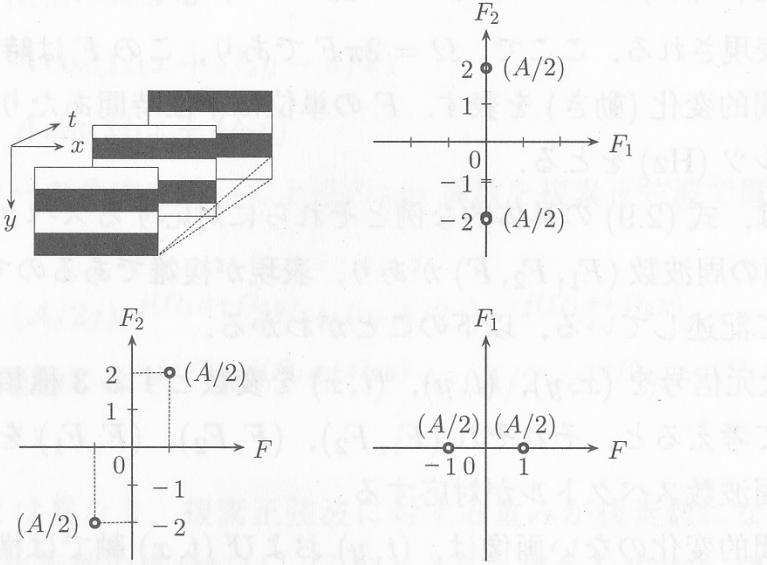
- Assume $\Omega = 2\pi F$. What are F and Ω?

Specrtum of Video

- What is the type of signal (a) ?
- What is the type of signal (b) ?



(a) $F_1 = 2$, $F_2 = 1$, $F = 0$



(b) $F_1 = 0$, $F_2 = 2$, $F = 1$

図 2.5 $g_a(x, y) = A\cos(2\pi(F_1 x + F_2 y + Ft))$

Exercise Example

- Specify 2D continuous signals ga(x,y) given by the spectral figures below.
- Illustrate black and white sketches of the above 2D signals.

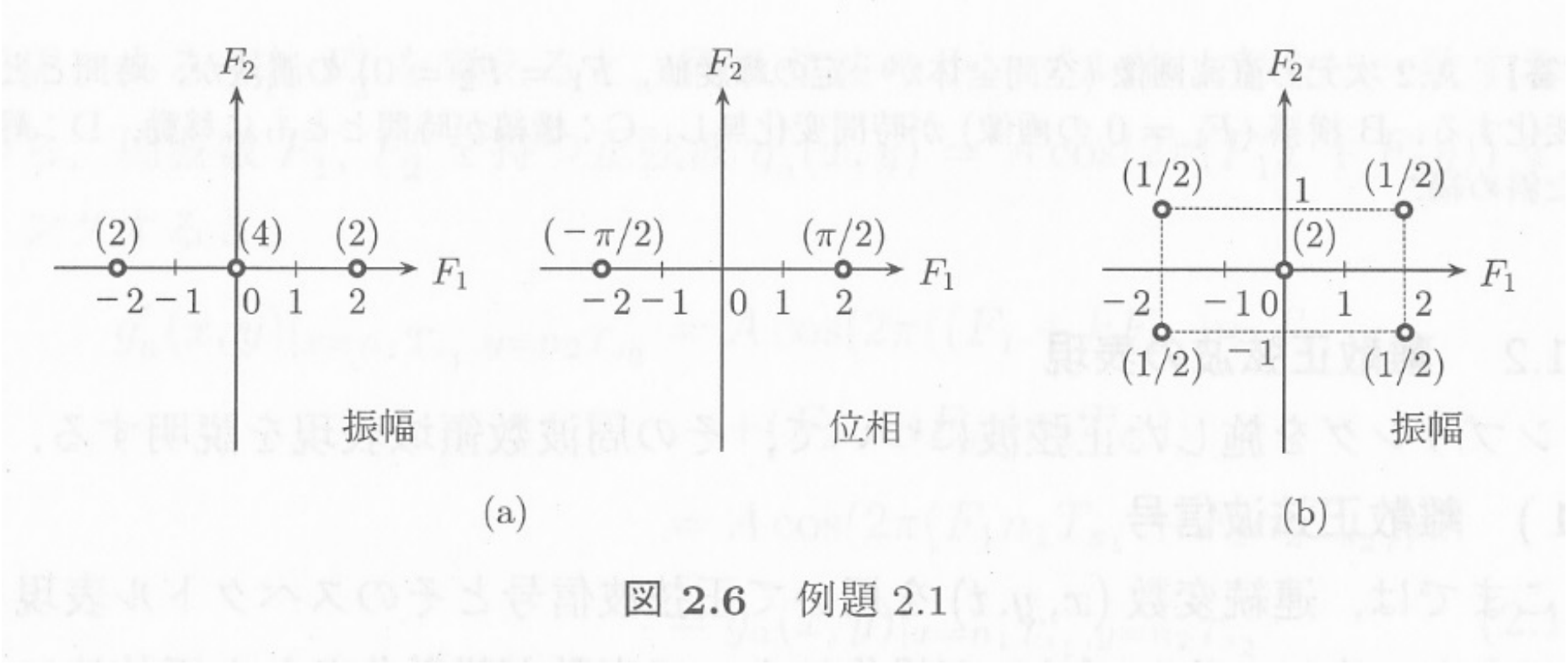


図 2.6　例題 2.1

Answers

- (a)
  $g_a(x,y)=4\times1+2\times\exp(j\pi/2)\exp(j2\pi2x)+2\times\exp(-j\pi/2)\exp(-j2\pi2x)$
  $=4+2\times2\times\sin(-4\pi x)$
  $=4-4\times\sin(4\pi x)$
  $=4+4\cos(4\pi x+\pi/2)$
- (b)
  $g_a(x,y)=2\times1+0.5\times\exp(j2\pi(2x+y))+0.5\times\exp(-j2\pi(2x+y))+0.5\times\exp(j2\pi(2x-y))+0.5\times\exp(-j2\pi(2x-y))$
  $=2+\cos(2\pi(2x+y))+\cos(2\pi(2x-y))$
  $=2+2\cos(4\pi x)\cos(2\pi y)$

Answer

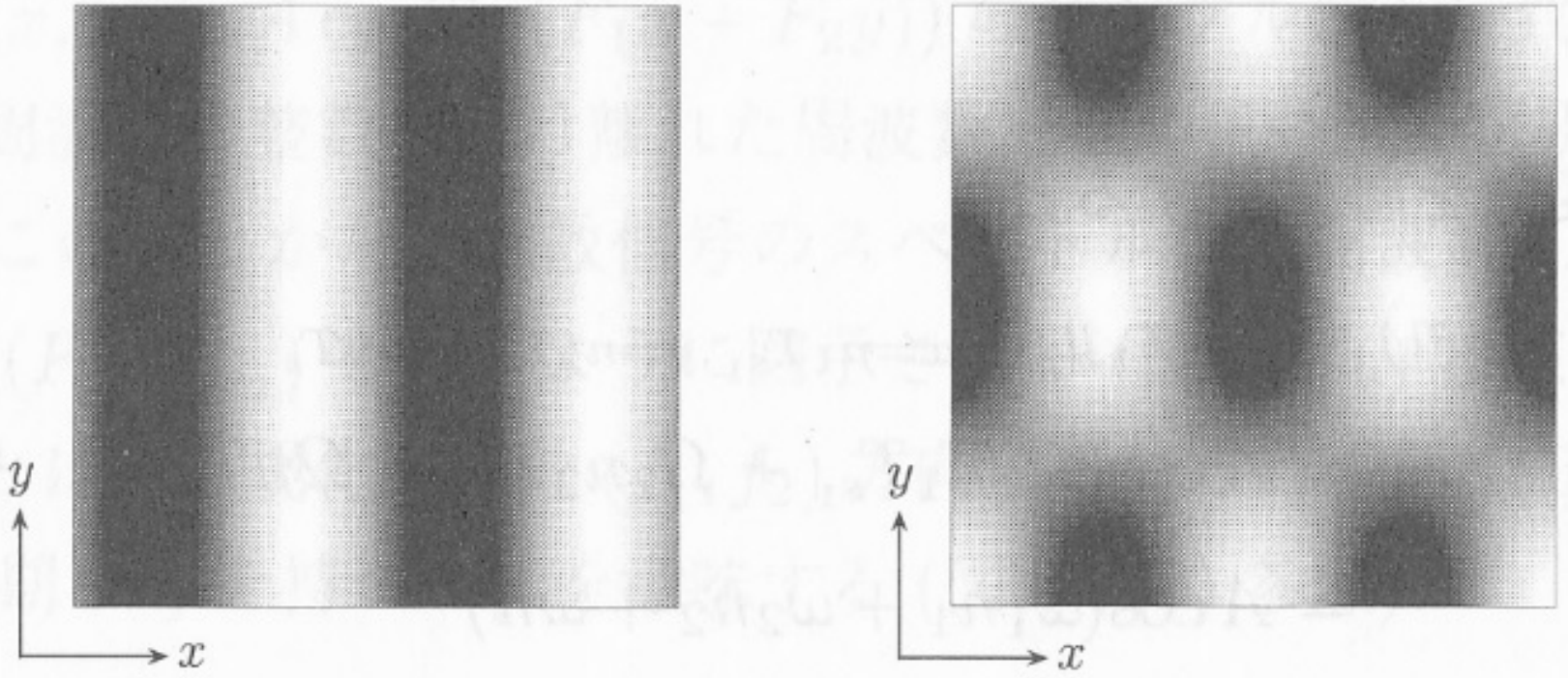


図 **2.7**　例題 2.1 の解答

Exercise Example

- What kinds of images correspond to 3D spectra A, B, C, and D?

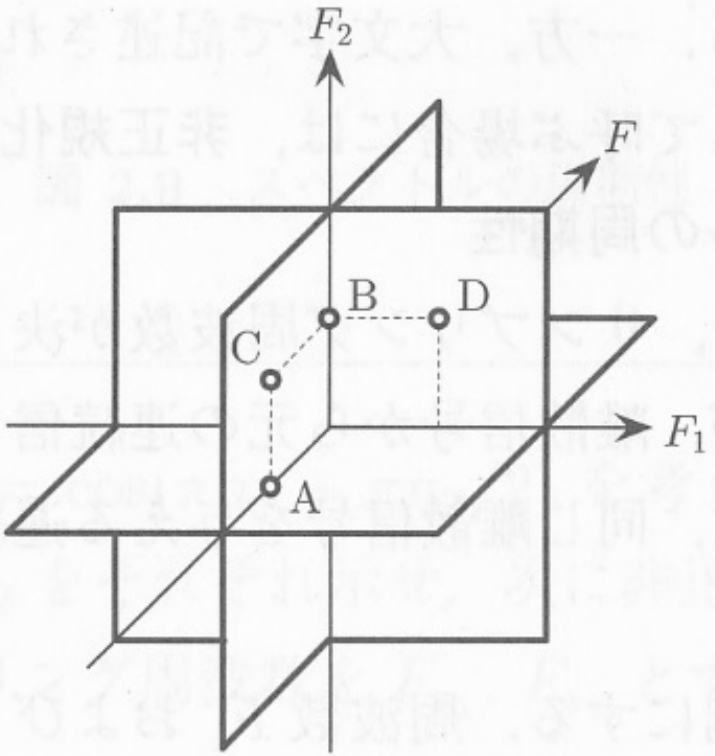


図 2.8　例題 2.2

Answers

- A : 2D direct current Image（constant intensity for all pixels, $F_1=F_2=0$）which changes the intensity temporally.
- B : still horizontal stripe（image with $F_1=0$）
- C : horizontal stripe（ image with $F_1=0$）moving vertically with time
- D : Slanted stripe unchanging with time