# Advanced Information Engineering

#2 October 24 (Mon), 2022

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# Image Signals

- Image information, in many cases, is transformed to [redacted] or [redacted] and is treated as image signals.

# Generation of Image Signals

- Light is reflected on subjects.
- Reflected light is [redacted].
- Image signals are recorded on [redacted], or transformed to electrical signals by [redacted].

[redacted]

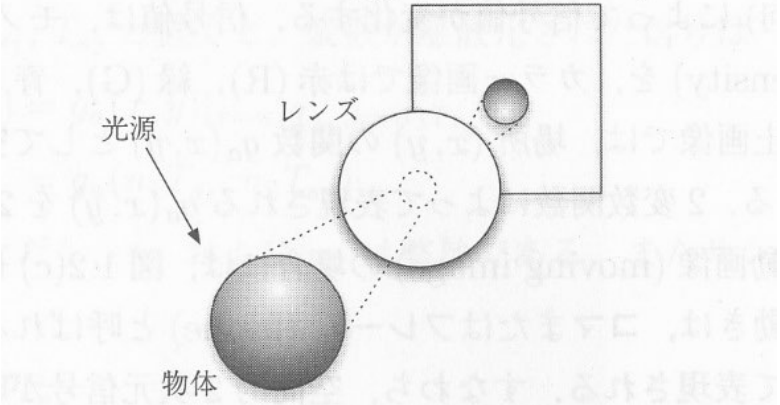
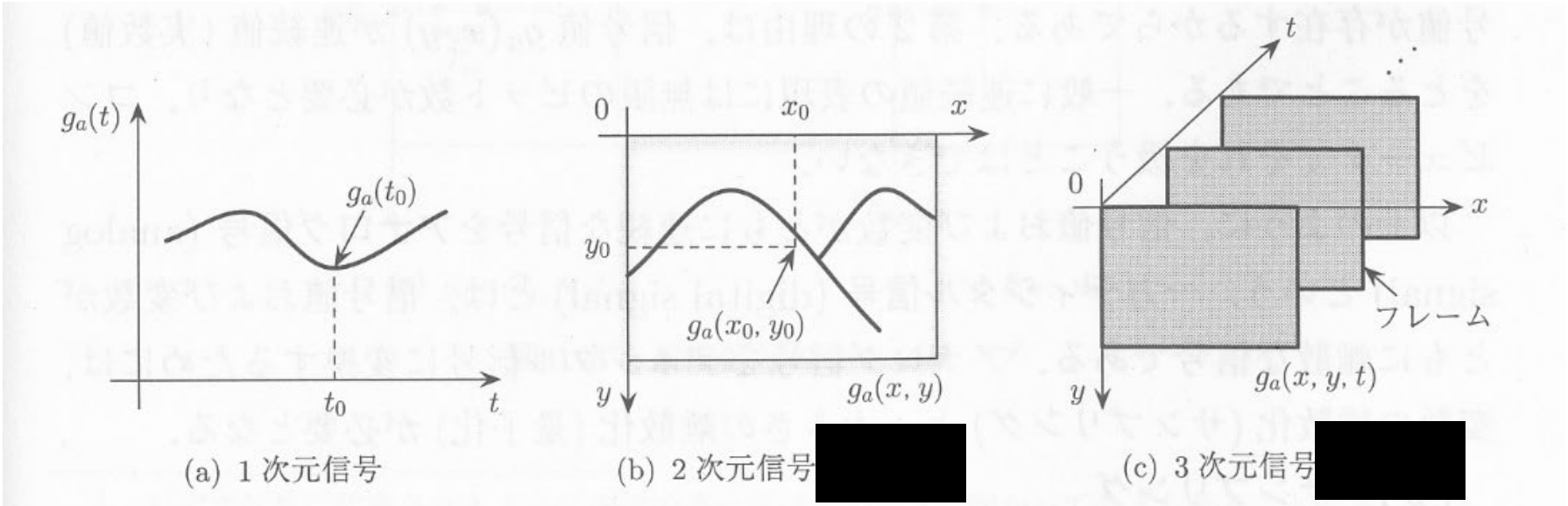



図 1.1　光学系による画像信号の生成

# Dimension of Signals

- Signals, whose typical example is [redacted], is a function of time $g_a(t)$ of one dimensional signal.
- [redacted] is two dimensional signals and has various values at spacial position (x,y) .




(a) 1 次元信号　(b) 2 次元信号[redacted]　(c) 3 次元信号[redacted]

図 1.2　信号の次元

# Digital Image

- Images taken by a digital camera are digital images and they are recorded as digital signals.
- Image processing processes digital signals by computers.

# Analog Image Signals

- Intensities of [redacted] are analog signals and are given by two dimensional signal $g_a(x,y)$.
- Variables x and y are [redacted] and [redacted] which is a discrete space point is not defined.
- Signal values $g_a(x,y)$ are [redacted] and are not handled by computers because continuous values need infinite number of bits.

# Digital Image Signals

- Image signals whose signals themselves g(x,y) and variables x, y are discrete.
- In order to convert analog signals to digital signals, it is necessary to discretize variables [redacted] and signals [redacted].

# Sampling

- Process to discretize variables x and y.
- Let $T_{s1}$, $T_{s2}$ be sampling intervals in x and y directions, respectively. The signal is given by

$$g(n_1, n_2) = g_a(x, y)|_{x=n_1 T_{s_1}, y=n_2 T_{s_2}}$$
$$= g_a(n_1 T_{s_1}, n_2 T_{s_2})$$

- $n_1$, $n_2$ are integers and reciprocals of sampling intervals Ts1, Ts2 are horizontal and vertical [redacted].
- A sample point in space is a [redacted] and its signal value is [redacted]

# [redacted] Sampling

- It is commonly used because of simpleness of output devices and processing.
- The top-left corner is the image origin (0,0) and $n_1$ and $n_2$ are variables in the horizontal and vertical directions.

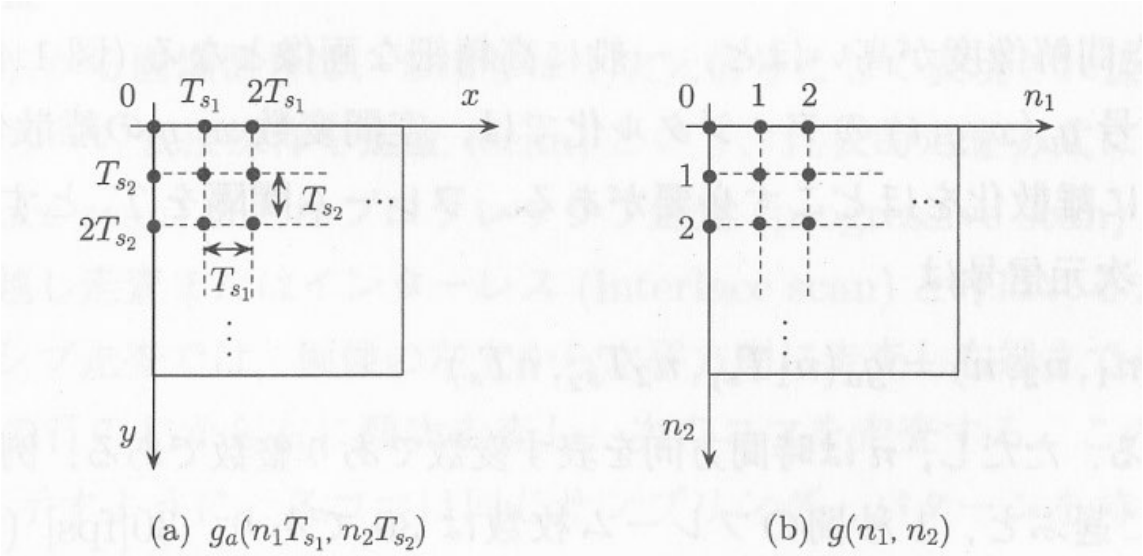



図 1.3　画像信号のサンプリング

# [redacted] of Image

- For a digital image, let $N_1$, and $N_2$ be pixel numbers in the horizontal and vertical axes. Its space resolution is given by $N_1 \times N_2$.
- Higher space resolution, higher-definition
- VGA, HD, 4K ?

図 1.4　解像度の異なる画像例

# Digitization of Video Image Signal $g_a(x,y,t)$

- Discretize time variable t by frame interval $T_s$.

$$g(n_1, n_2, n) = g_a(n_1 T_{s_1}, n_2 T_{s_2}, n T_s)$$

- n is a variable for time elapse and is an integer.
- For example if $T_s$ = 1/30[sec], [redacted], which corresponds to time resolution is 30fps.
- Fs, the reciprocal of $T_s$ $F_s$ is called time sampling frequency.

# Sample Exercise

- Let's think about two one-dimensional time signals of frequency F=1[Hz] and F'=5[Hz], $g_a(t)=\cos(\blacksquare t)$ and $g_a(t)=\cos(\blacksquare t)$. Please illustrate discrete signals obtained by sampling these signals with sampling interval $T_s=1/F_s=1/4$[sec] .

# Answers

- Their sampled values are the same.
- Sampling time $t=nT_s$, and
  $g(n)=\cos(\pi n/2)=\cos(2\pi+\pi n/2)=g'(n)$
- Generally when $F'=F+kF_s$ (k is an integer), both of the sampled values become identical.

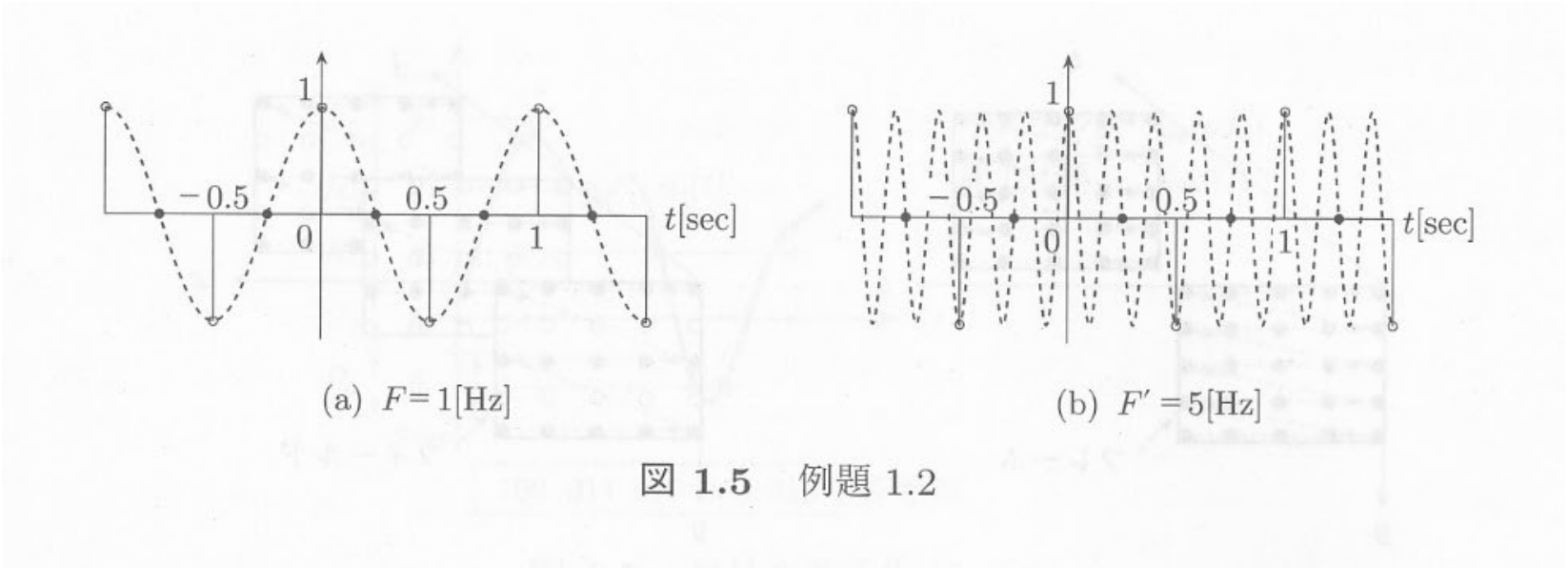



図 1.5　例題 1.2

# [redacted](走査)

- Process to transform multi-dimensional signals to one dimensional signals.

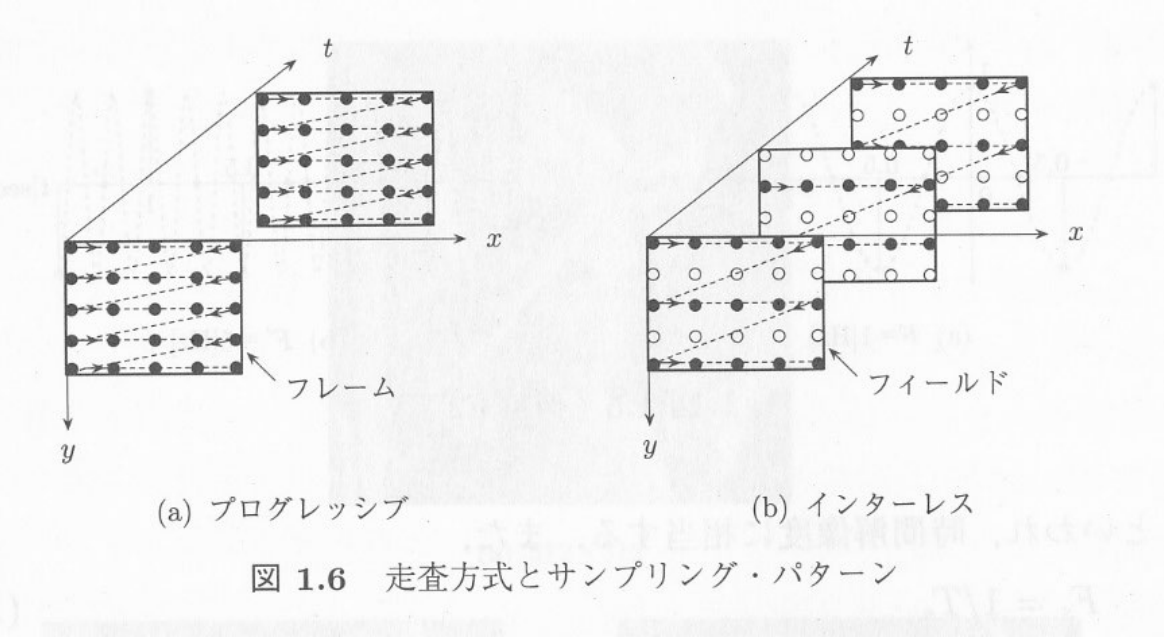



図 1.6　走査方式とサンプリング・パターン

- TV broadcasting uses [redacted] and movie files do [redacted].
- Prioritize time resolution or space resolution?

# Digital Image Signal（review)

- Image signal g(x, y) and variables x, y are discrete.
- To convert analogue signal to digital signal, discretization of variables ([redacted]) and that of image signal ([redacted]) are necessary.

# 量子化([redacted])

- Because analog signal are real numbers and infinite,  for quantization, for example with 8 bits it must be replaced with values $L=2^8=256$.
- Quantization means that values of some number are replaced with other values of smaller number.

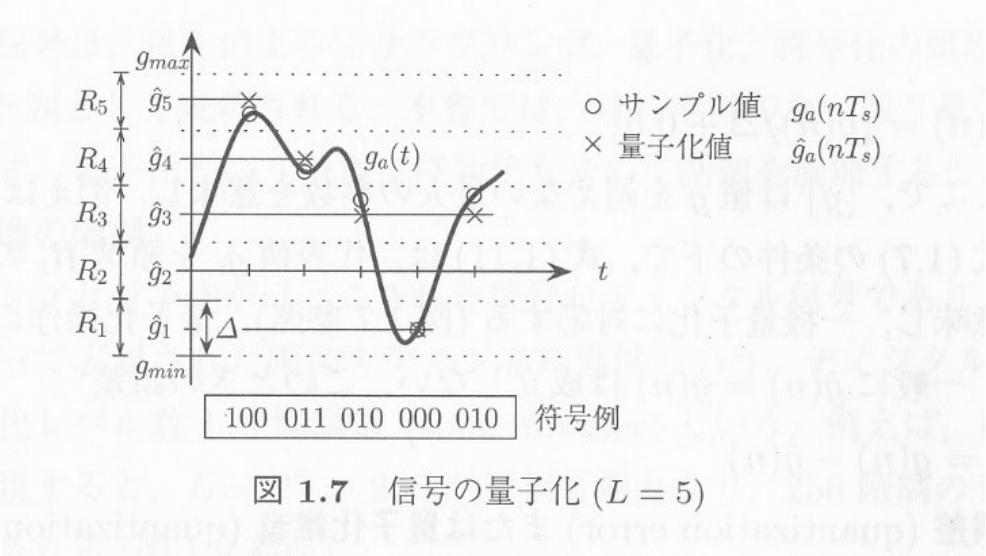



図 1.7　信号の量子化 ($L = 5$)

# 量子化(quantization)

- L is called quantization level or quantization step number.
- The difference between maximum and minimum values $g_{max}$ - $g_{min}$ is called [redacted].

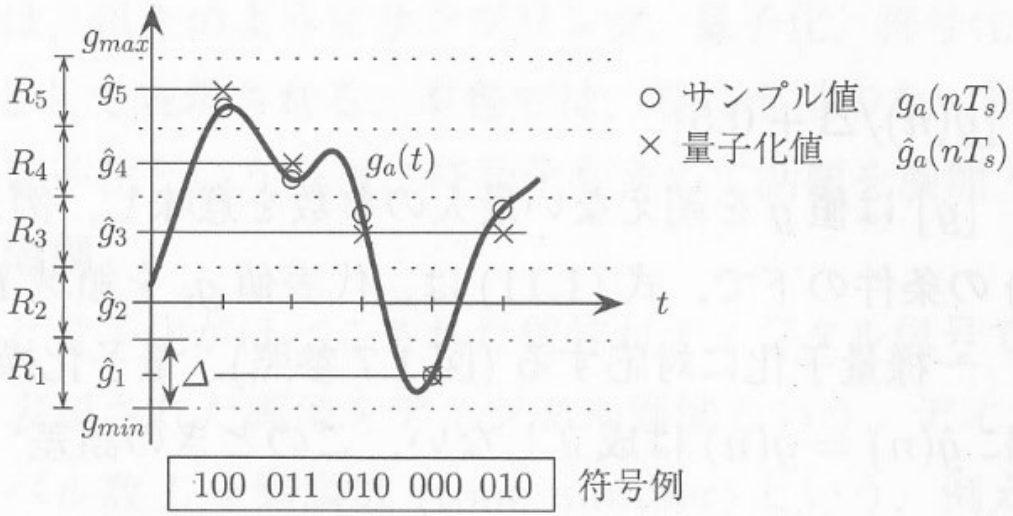



図 1.7 信号の量子化 ($L = 5$)

# 量子化(quantization)

- By quantization step number L and dynamic range $g_{max}$ - $g_{min}$,

  quantization step $\Delta=(g_{max} - g_{min})/L$

  If Δ dose not depend on the index, it is

  called uniform quantization.
- In case where a sample number is a neighborhood of a representative value, it is regarded as quantization value.

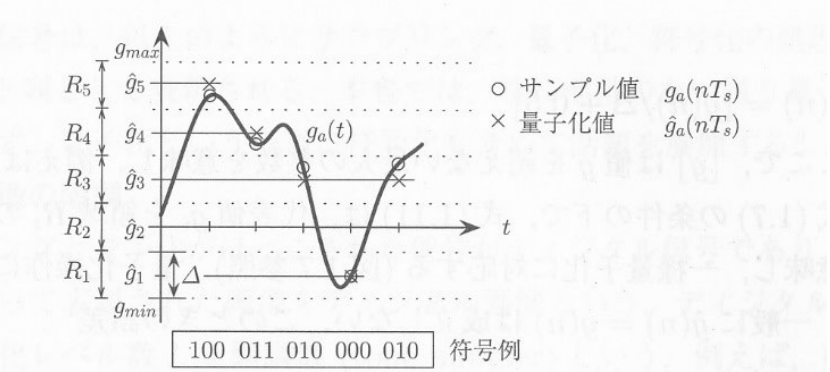



図 1.7 信号の量子化 ($L = 5$)

# 量子化誤差(quantization [redacted])

- The difference between a sample value and its quantization value is called quantization [redacted] or quantization noise.
- The range of quantization error e is given with quantization step Δ by

$$-\Delta/2 \leqq e < \Delta/2$$

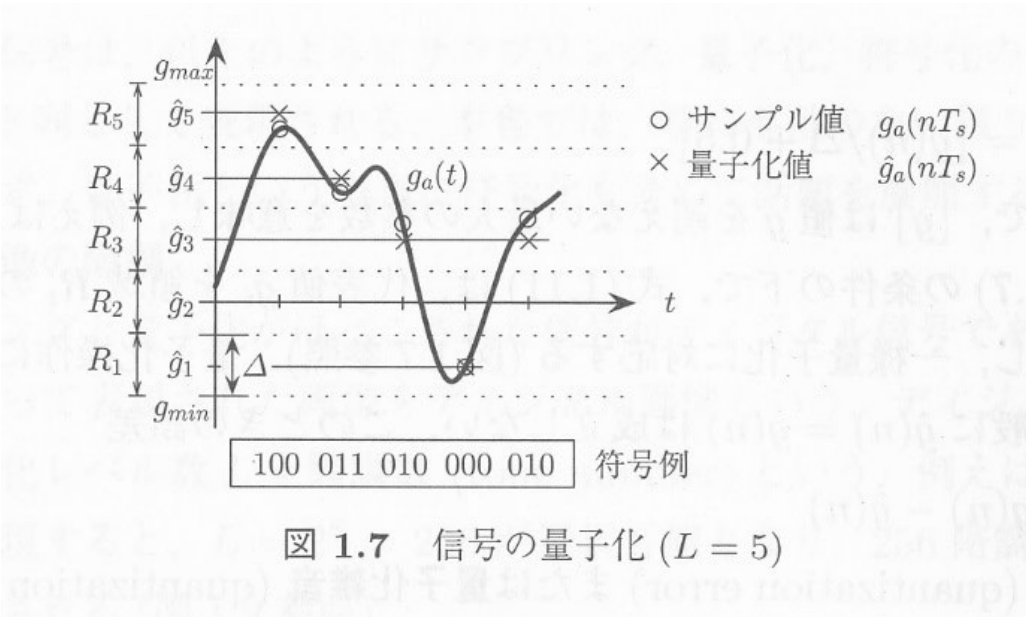



図 1.7 信号の量子化 ($L = 5$)

# 量子化誤差(quantization error)

- It is possible to make quantization error smaller by [redacted], but [redacted] are needed.

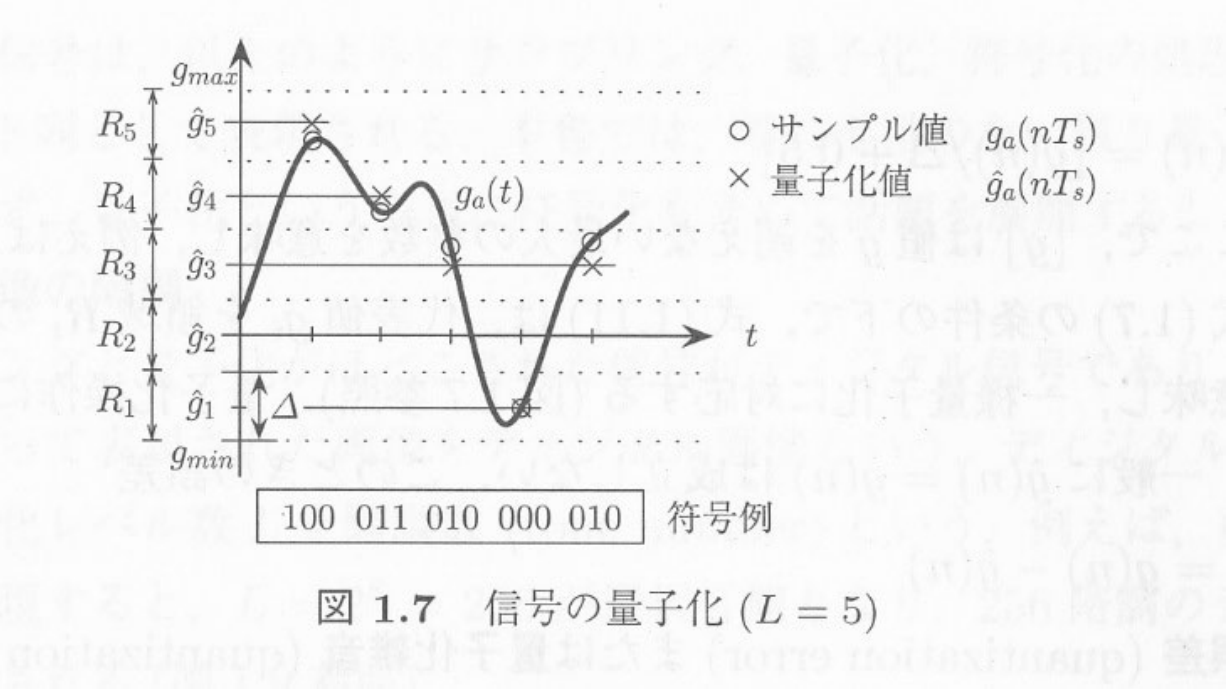



図 1.7　信号の量子化 ($L = 5$)

# Encoding

- Process where signal values quantized with quantization level L are assigned to integer index $n_i$ (i=1,2,・・・,L) which are one to one correspondent with the signal values.

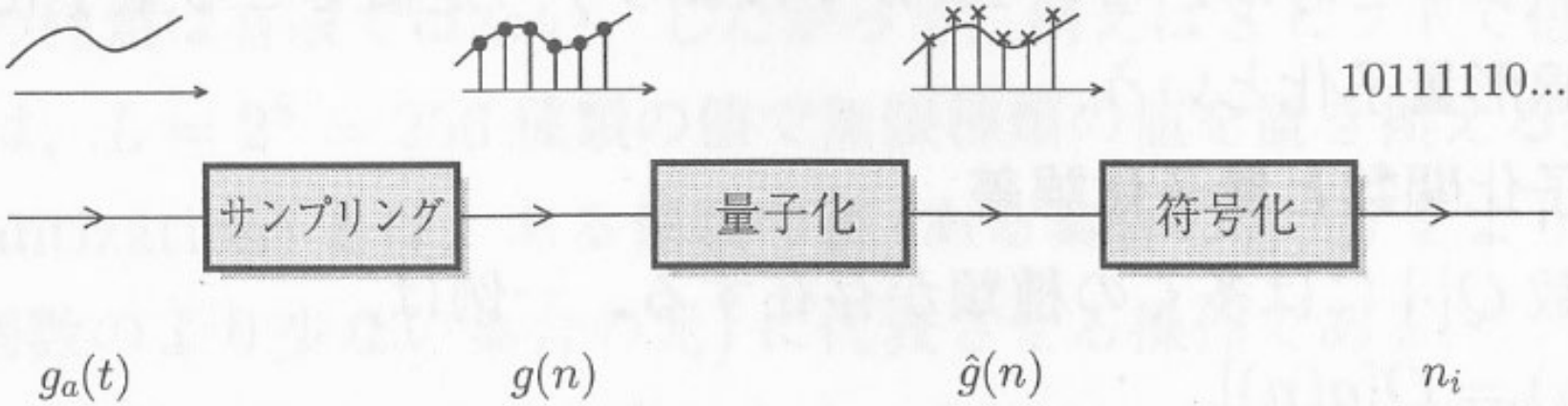



図 1.8　サンプル値の符号化

# 符号化([redacted])

- In case where L=5, [redacted] are used for encoding.
- Encoding with [redacted] is given as binary numbers by [redacted]

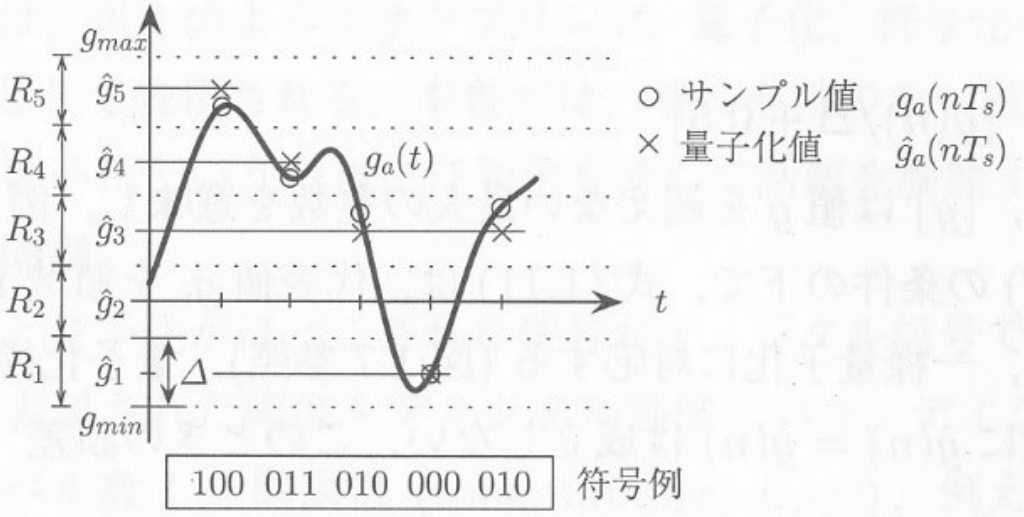



図 1.7　信号の量子化 ($L = 5$)

# Digital Image

- The digital signal is an analog signal modified by [redacted] and [redacted] and the image represented by a digital signal is called digital image.

# Tone of Image

- For the digital image, quantization level number is called tone.
- What is the tone of 16-bit image ?

(a) 2 階調(1 ビット)

(b) 4 階調(2 ビット)

(c) 256 階調(8 ビット)

図 1.9　階調の異なる画像例

# 画像の階調([redacted])

- The pixel value of a digital image is generally a positive integer.

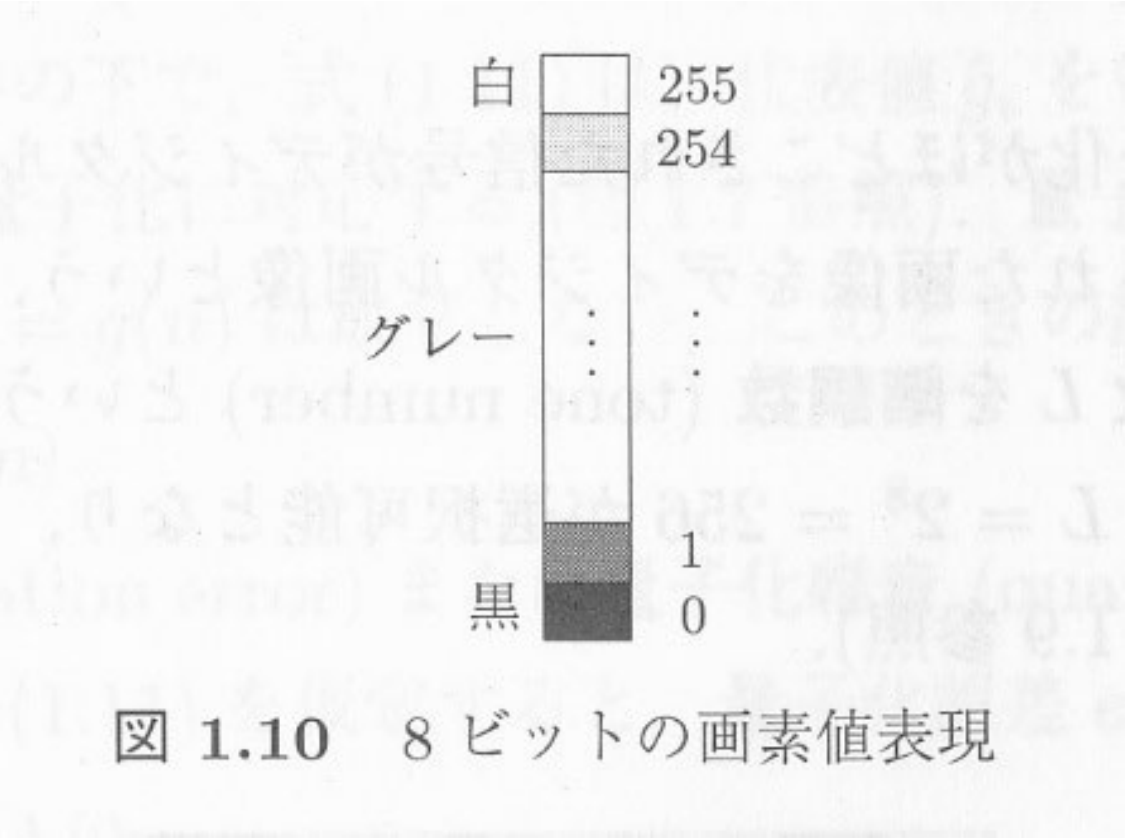



図 1.10　8 ビットの画素値表現

# Bit-rate

- Transmission velocity of data transmission
- Its unit is bps.
- The bit-rate $B_r$ is given by

$$B_r = N_1 \times N_2 \times F_s \times l$$

for a gray-scale video with spatial resolution $N_1 \times N_2$, frame-rate $F_s$, tone $L = 2^l$.

- For color video, its bit-rate is 3 times larger.

# Exercise #1

- Assume that we use uniform quantization and a sample value g(n)=7. Quantize the value with quantization step Δ=5 and Δ=2,respectively. Note that we are supposed to use rounding (四捨五入). Calculate quantization value and quantization value.

# Answer

- When quantization step $\Delta=5$, quantization value = 5, quantization error = 2.
- When quantization step $\Delta=2$, quantization value = 8, quantization error -1.

# Exercise #2

- We would like to quantize a signal whose dynamic range $g_{max} - g_{min} = 10$ and make quantization error be less than or equal to ¼. How many bits are necessary to encode quantization values?

# Answer

- The range of quantization error

$$-\Delta/2 \leqq e < \Delta/2$$

- Hence quantization step Δ should be ½.
- Then quantization step number L = 10/(1/2)=20.
- Since L=20≦$2^5$, 5 bits are necessary.

# Exercise #3

- Calculate bit-rate $B_r$ for color video with its spacial resolution 1000×1000, frame-rate 30 fps, 8-bit tone. If necessary, please use K (kilo), M (mega), G (giga).

# Answer

- $B_r = 1000 \times 1000 \times 30 \times 8 \times 3 = 720$[Mbps]