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Aesthetic Spiral for Design

(Lingkaran Estetik untuk Reka Bentuk)

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ABSTRACT

A planar spiral called Generalized Log Aesthetic Curve segment (GLAC) has been proposed using the curve synthesis process with two types of formulation; ϱ-shift and ϰ-shift. Both methods were carried out by extending the formulation of Generalized Cornu Spiral (GCS) in a similar manner to the Log Aesthetic Curve (LAC). The family of GLAC comprises of planar curves of high quality such as GCS, LAC, clothoid, logarithmic spiral and circle involute. The GLAC segment has an additional parameter to determine its shape as compared to GCS and LAC segment, hence an extra constraint can be satisfied when shaping the GLAC segment. The last section of the paper shows a numerical example.

Keywords: Curve synthesis; high quality curves; monotonic curvature curves; spiral

ABSTRAK

Satu lingkaran segmen 2D bernama Lengkung Astetik Log Umum (GLAC) telah dibentuk dengan menggunakan proses sintesis lengkung melalui dua jenis perumusan; ϱ-shift dan ϰ-shift. Kedua-dua rumusan ini dikembangkan dengan menggunakan rumus Lingkaran Cornu Umum (GCS) yang mirip dengan pembentukkan rumus Lengkung Astestik Log (LAC). Ahli keluarga GLAC ini terdiri daripada lengkung-lengkung yang berkualiti tinggi contohnya seperti GCS, LAC, lingkaran klotoid, lingkaran logarithma, bulatan involut dan sebagainya. Segmen GLAC ini juga mempunyai satu parameter tambahan untuk menentukan bentuknya berbanding dengan segmen GCS dan LAC, maka satu kekangan tambahan dapat dipenuhidalam pembentukan segmen GLAC. Bahagian akhir menunjukkan satu contoh berangka.

Kata kunci: Lengkung berkualiti tinggi; lingkaran; lengkung berkelengkungan monotonik; sintesis lengkung

INTRODUCTION

A potential customer judges the aesthetic appeal of products before the physical performance (Pugh 1991). This clearly indicates the importance of aesthetic shapes for the success of industrial products. High quality curves are also known as aesthetic curves and other terms that have been used widely to describe these types of curves include minimal energy curves. The main characteristic of these curves is that it has a monotonic curvature profile. A mathematical term used to describe the planar curves with monotonic (increase or decrease) of curvature is called a spiral (Guggenheimer 1963). Fair curves or also known as beautiful curves consist of curves having few monotonic curvature profiles (Farin 2002).

Similarly, a curve is characterized based on its curvature profile in the curve design environment. A curvature profile is a graph plotted with the values of parameter t representing x-axis against its corresponding signed curvature values representing y-axis (Nutbourne & Martin 1988). There are many studies indicating the importance of the curvature profile to characterize planar curves (Nutbourne & Martin (1988); references therein). The designer arrives to the desired shape by interactively tweaking the control points and concurrently inspecting the curvature plot. In practice, a curvature profile has been highlighted as a shape interrogation tool to fair B-spline curves and surfaces (Buchard et al. 1994; Farin & Sapidis 1989).

A different kind of approach has been proposed by Harada et al. (1994) to analyze the characteristics of planar curves used for automobile design. The relationship between the length frequency of a segmented curve with regards to its radius of curvature is plotted in a log-log coordinate system and called Logarithmic Distribution Diagram of Curvature (LDDC). To note, the generation of a LDDC is through quantitative methods. Kanaya et al. (2003) proposed the generation of logarithmic curvature graph (LCG) to substitute LDDC. LCG is an analytical way of obtaining the relationship between the interval of radius of curvature and its corresponding length frequency. According to Harada et al. (1994) and Kanaya et al. (2003), the detection of aesthetic curves is by inspecting the gradient of LCG whereby if it is constant then the curve is categorized as aesthetic. Miura (2006a) derived the explicit formulation of LCG and further developed a family of curves having constant gradient of LCG called Log Aesthetic Curve (LAC) (Muira 2006b). Yoshida and Saito (2006) extended LAC to produce an interactive LAC segment in which a designer can shape the curve by tweaking three control points similar to the Bezier quadratic curve. In the following year, they proposed quasi-aesthetic curves in rational cubic Bezier forms (Yoshida & Saito 2007).

Quasi-aesthetic curves have monotonic curvature profile with almost constant LCG gradient. Hence, further analysis is needed to identify a general family of aesthetic curves which was carried out by Gobithaasan et al. (2008 & 2009).

Gobithaasan et al. (2008) derived a general formulation of LCG gradient and they further indicated that a linear LCG gradient function is a better condition to coin aesthetic curves based on the analysis of Generalized Cornu Spiral (GCS) (Gobithaasan et al. 2009). Furthermore, they indicated that the idea of aesthetic curves having constant LCG gradient is too restrictive as linear LCG gradient is a general form and constant LCG is just one simple case. GCS (Ali et al. 1999) was chosen as it has been used to produce high quality surface via point based CADCAM (Cripps 2003).

The family of LAC and GCS curve segments has different features regardless of their monotonic curvature functions. The family of LAC has constant LCG gradient values and the family of GCS consists of a linear LCG gradient function. Furthermore, the LCG gradient function of GCS can only be constant when it becomes either clothoids or logarithmic spirals.

This paper shows the approaches used to obtain a general formulation representing aesthetic curves. The final result is a new kind of curve called Generalized Log Aesthetic Curve (GLAC) (pronounced as 'G-LAC'). The main feature of this curve is that its curvature function is always monotonic and the family consist of clothoid, logarithmic spiral, circle involute, Neilsen's curve, LAC and GCS. It has an extra degree of freedom which can be used to satisfy constraints that may occur during constructing design models.

LOGARITHMIC CURVATURE HISTOGRAM

Consider a planar curve given as *C(t)* and the first derivative of LCG for *C(t)* exists. Let *s(t)* and *ρ(t)* be its arc length and radius of curvature function respectively, then the LCG for *C(t)* can be obtained using (Miura 2006a):

$$LCH(t) = \left\{ Log\left[\rho(t)\right], Log\left[\frac{\rho(t)s'(t)}{\rho'(t)}\right]\right\}, \tag{1}$$

and the LCG gradient is obtained using the following equation (Gobithaasan et al. 2009):

$$\lambda(t) = 1 + \frac{\rho(t)\rho''(s)}{\rho'(t)^z}\left(\frac{\rho'(t)s''(t)}{s'(t)} - \rho''(t)\right). \tag{2}$$

If the curve is parameterized by arc length *s*, then (2) can further be simplified as follows:

$$\lambda(s) = 1 - \frac{\rho(s)\rho''(s)}{\rho'(s)^2}. \tag{3}$$

DEVELOPMENT OF ρ-SHIFT & κ-SHIFT GLAC

The underlying principal of developing LAC is by forcing the curves to preserve $\lambda_{LAC}(s) = \alpha$ where $\alpha \in$ and satisfying the initial conditions at the origin where $\rho_{LAC}(0) = 1$ and $\theta_{LAC}(0) = 0$. $\rho_{LAC}(s)$ is formulated in the form of Cesaro equation (Yoshida & Saito 2006) as follows:

$$\rho_{LAC}(s) = \begin{cases} e^{\Lambda s}, & \text{if } \alpha = 0 \\ (\Lambda \text{as} + 1)^{1/\alpha}, & \text{otherwise} \end{cases}, \tag{4}$$

where ***e*** representing the exponential function and $\{\Lambda, \alpha\} \in \mathbb{R}$ are shape parameters. The next section shows two types of formulation of GLAC.

ρ-SHIFT GLAC

The first formulation of GLAC is derived by manipulating the GCS's radius of curvature and the result is a *ρ*-shift GLAC radius of curvature function as $\rho_{\widehat{GLAC}}(s) = (\Lambda \text{as} + 1)^{1/\alpha} + \upsilon$. Hence, the curvature function can be written as stated in (5) (for simplicity, the case when α = 0 is omitted):

$$\kappa_{\widehat{GLAC}}(s) = \frac{1}{(\Lambda\alpha s + 1)^{1/\alpha} + \upsilon}. \tag{5}$$

The directional angle of *ρ*-shift GLAC is as follows:

$$\theta_{\widehat{GLAC}}(s) = \theta_0 + \int_0^s \frac{1}{(\Lambda\alpha a + 1)^{1/\alpha} + \upsilon}\,\text{du}. \tag{6}$$

Finally the parametric form of the *ρ*-shift GLAC is as follows:

$$C_{\widehat{GLAC}}(s) = P_0 + \left\{\int_0^s \cos\left[\theta_{\widehat{GLAC}}(u)\right]du, \int_0^s \sin\left[\theta_{\widehat{GLAC}}(u)\right]du, \tag{7}$$

where P_0 is the given beginning point, θ_0 is the angle between the tangent at the beginning point which is measured anti-clockwise from the x-axis, and $\{\Lambda, \alpha, \upsilon\} \in \mathbb{R}$ are variables that can be used shape the GLAC segment.

κ-SHIFT GLAC

Another way to formulate the GLAC is by manipulating (4) directly to obtain (8) and for simplicity case α = 0 is omitted,

$$\kappa_{\widetilde{GLAC}}(s) = (\Lambda\alpha s + 1)^{-1/\alpha} + \upsilon. \tag{8}$$

Similar steps shown in the previous section are carried out to obtain its directional angle function and κ-Shift GLAC. The direction angle *θ*(*s*) of a κ-shifted GLAC is given by:

$$\theta_{\widetilde{GLAC}}(s) = \theta_0 + \int_0^s (\Lambda\alpha u + 1)^{-1/\alpha} + \upsilon\,\text{du}. \tag{9}$$

Unlike the formulation of GLAC using ρ-shift, the above equation can be integrated analytically as shown in (10), thus numerical integration is not necessary to calculate the direction angle:

$$\theta_{\widetilde{GLAC}}(s) = \theta_0 + \frac{(\Lambda\alpha s+1)^{\alpha-1/\alpha} - 1}{(\alpha-1)\Lambda} + \upsilon s. \quad (10)$$

The parametric form of the planar κ-shift GLAC is given by the following form:

$$C_{\widetilde{GLAC}}(s) = P_0 + \left\{ \int_0^s \cos\left[\theta_{\widetilde{GLAC}}(u)\right] du, \int_0^s \sin\left[\theta_{\widetilde{GLAC}(u)}\right] du \right\}, \quad (11)$$

where P_0 is the given beginning point, θ_0 is the angle between the tangent at the beginning point which is measured anti-clockwise from the x-axis, and $(\Lambda, \alpha, \upsilon) \in \mathbb{R}$ are variables that can be used shape the GLAC segment.

Since, the formulation of κ-shift GLAC is better than ρ-shift GLAC, κ-shift GLAC will be used to construct the general parametric representation of GLAC for any arbitrary α which satisfies the conditions at the origin similar to LAC; $\rho_{\text{GLAC}}(0) = 1$ and $\theta_{\text{GLAC}}(0) = 0$.

GLAC SATISFYING CONSTRAINTS AT THE ORIGIN

The curvature of GLAC is formulated to be more flexible than LAC in which designers may define the curvature at the origin of the curve.

$$\kappa_{GLAC}(s) = \begin{cases} e^{-\Lambda s} + \upsilon, & \text{if } \alpha = 0 \\ (\Lambda\alpha s+1)^{-1/\alpha} + \upsilon, & \text{otherwise} \end{cases}. \quad (12)$$

In order to verify that GLAC can be controlled using at the starting point, one may simplify equation (12) by letting $s=0$ to obtain the following equation:

$$\rho_{GLAC}(0) = \frac{1}{1+\upsilon} \quad (13)$$

Equation (12) can further be expressed in the arc length function $s(\rho)$ as follows:

$$S_{GLAC}(\rho) - \begin{cases} \frac{1}{\Lambda} Log\left[\frac{1}{\rho^{-1} - \upsilon}\right], & \text{if } \alpha = 0 \\ \frac{(\rho^{-1} - \upsilon)^{-\alpha} - 1}{\Lambda\alpha}, & \text{otherwise} \end{cases}. \quad (14)$$

Since $\frac{d\theta}{ds} = \frac{1}{\rho}$, (12) is integrated to obtain the directional angle of GLAC. The case α = 1 is isolated in order to obtain a suitable $0_{GLAC}(s)$ (function. The following directional angle is formulated by forcing $\theta_{GLAC}(0) = 0$,

$$\theta_{GLAC}(s) = \begin{cases} \frac{1}{\Lambda}\left(1 - e^{-\Lambda s}\right) + \upsilon s, & \text{if } \alpha = 0 \\ \frac{1}{\Lambda} Log\left[\Lambda s + 1\right] + \upsilon s, & \text{if } \alpha = 1 \\ \frac{(\Lambda\alpha s+1)^{\frac{\alpha-1}{\alpha}} - 1}{\Lambda(\alpha-1)} + \upsilon s, & \text{otherwise} \end{cases}. \quad (15)$$

With these findings, a compact GLAC equation is derived (Gobithaasan 2010):

Theorem 1: *The parametric form of a GLAC segment is given by:*

$$C_{GLAC}(s) = \left\{ \int_0^s \cos\left[\theta_{GLAC}(u)\right] du, \int_0^s \sin\left[\theta_{GLAC}(u) du\right], \right. \quad (16)$$

where e represents the exponential function and $\{\Lambda, \alpha, \upsilon\} \in \mathbb{R}$ are variables which can be used for shaping the GLAC segment. This curve begins at the origin and the selection of the directional angle function $\theta_{GLAC}(u)$ depends on the value of α as stated in (16).

NUMERICAL EXAMPLES

This section illustrates a family of GLAC segments when α = 1; with Λ = 1 and υ = {-0.1, -0.05. 0, 0.05, 1} for $0 \leq s \leq 5$. Figure 1 illustrates the family of curves obtained with the stated data followed by Figures 2 and 3 indicating its corresponding LCG and $\lambda(s)$ (plots, resepectively. The shape of the GLAC segment is similar to GCS segment when the GLAC segment produces linear LCG gradient. A circle involute is produced when ν = 0. However, the GLAC formulation has three shape parameters; α, Λ and υ giving more flexibility for the designers to produce the desired shapes. To note, the GLAC's family is huge as it can even produce exponential type of LCG gradient yet preserve the monotonicity of its curvature; hence this curve is guaranteed to be visually pleasing.

CONCLUSION

Two methods to construct a GLAC segment has been shown; the ρ-shift and the κ-shift method. Both methods are the extensions of the GCS formulation in a similar manner to the LAC segment. As a result, a family of GLAC segment for aesthetic design has been vastly increased. The GLAC segment has an additional parameter to determine its shape as compared to the LAC segments; hence an extra constraint can be satisfied during shaping the GLAC segments giving more flexibility. The formulation to control the GLAC segment via three control points is progressing well and expected to be a promising aid for the designers to produce visually pleasing products with less effort.

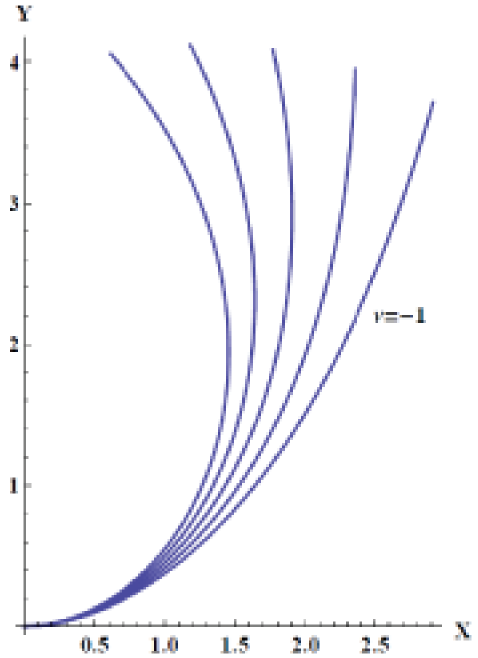

FIGURE 1. The family GLAC segments when $\alpha = 1$

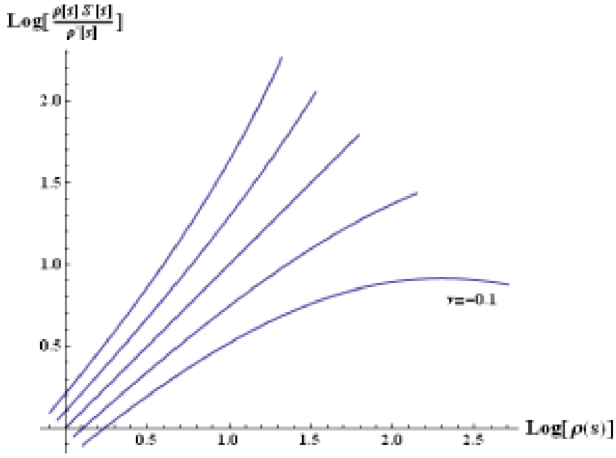

FIGURE 2. The corresponding LCG Plot of Figure 1

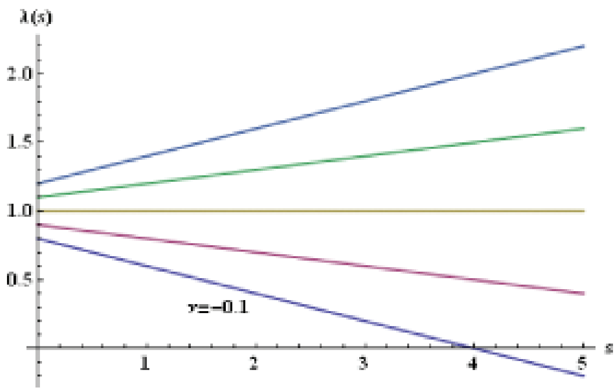

FIGURE 3. The corresponding $\lambda(s)$ plot of Figure 1

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