Input of Log-aesthetic Curves with a Pen Tablet

Kenjiro T. Miura, Mariko Yagi, Yohei Kawata, Shin'ichi Agari Makoto Fujisawa, Junji Sone*
Graduate School of Science and Technology
Shizuoka University
Address 3-5-1 Johoku, Hamamatsu, Shizuoka, 432-8561, Japan
*Department of Applied Computer Science
Tokyo Polytechnic University
e·mail: tmkmiur@ipc.shizuoka.ac.jp
www: http://ktm11.eng.shizuoka.ac.jp/profile/ktmiura/welcome.html

Abstract

The aesthetic curves include the logarithmic (equiangular) spiral, clothoid, and involute curves. Although most of them are expressed only by an integral form of the tangent vector, it is possible to interactively generate and deform them and they are expected to be utilized for practical use of industrial and graphical design. However, their input method proposed so far by use of three so-called control points can generate only an aesthetic curve segment with monotonic curvature variation and can not create a curve with the curvature-extremal point or the inflection point where the derivative of the curvature or the curvature itself changes its sign, respectively.

Hence we propose a method to generate an aesthetic curve with G^1 continuity from a sequence of 2D points input with, for example, a liquid cristal pen tablet.

Keywords: aesthetic curve, pen tablet

1 Introduction

For the sake of advance of the Information Technology, the technologies of the development and manufacturing of industrial products have been improved in terms of efficiency. However the advances of the technologies of many countries other than Japan are also significant and we need researches on the design pursuing beauty to differentiate Japanese products from those made in other countries. Since the design of industrial products heavily depends on the designers' feeling and sensitivity (kansei in Japanese), the communications between the designers and manufacturers are not always smooth. It is desirable to represent the designers' kansei as mathematical expressions as much as possible to solve this problem.

In this paper, we propose a method to generate "beautiful" curves as a technique to support the design pursuing beauty mentioned above. By use of the general equations of aesthetic curves, we intend to add the aesthetic property to the curves generated by approximating the curves directly drawn with, for example, a liquid-crystal pen tablet. The log-aestehtic curves were proposed by Harada et al.[1] as such curves whose logarithmic distribution diagram of curvature(LDDC) is approximated by a straight line. Miura[2, 3] derived analytical solutions of the curves whose logarithmic curvature graph(LCG):an analytical graph version of the LDDC are strictly given by a straight line and proposed these lines as the general equations of aesthetic curves. Furthermore, Yoshida and Saito[4] analyzed the properties of the curves expressed by the general equations and developed a new method to interactively generate a curve by specifying two end points and the tangent vectors there with three control points as well as the slope of the straight line of the LCG. In this research, we call the curves expressed by the general equations of aesthetic curves the log-aesthetic curves. Figure 1 shows several aesthetic curves with various α values: the slopes of the LCG.

The aesthetic curves include the logarithmic (equiangular) curve ($\alpha = 1$), the clothoid curve ($\alpha = -1$), the circle involute curve (α=2), and

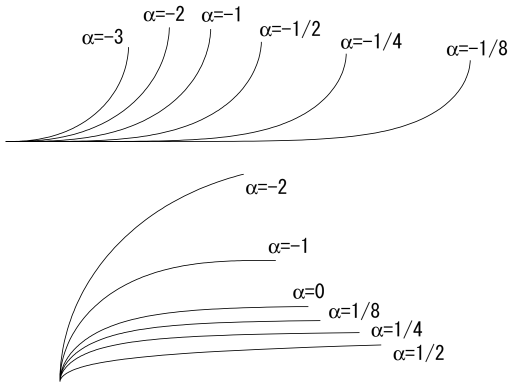


Figure 1: Aesthetic curves of various α's

Nielsen' spiral ($\alpha = 0$). It is possible to generate and deform the aesthetic curve even if they are expressed by integral forms using their unit tangent vectors as integrands ($\alpha \neq 1, 2$) and they are expected to be used in practical applications. However, their input method proposed so far by use of three so-called control points can generate only an aesthetic curve segment with monotonic curvature variation and can not create a curve with segments of different α values such as the compounded-rythm curve[1].

Therefore this paper proposes a method where at first we input a sequence of points as a curve by use of a liquid-crystal pen tablet, approximate it by a B-spline curve, subdivide the B-spline curve at inlection and curvature-extremal points, then again approximate each segment of the subdivided B-spline curve by a log-aesthetic curve to generate a curve with complex rhythm. Note that since the curvature of a log-aesthetic curve monotonically increases or decreases, we have to subdivide the B-spline curve at the curvature extremum.

2 Log-aesthetic Curve

2.1 Curve expressions

In this reseach, we define the log-aesthetic curve as the curve whose logarithmic curvature graph is stricly expressed by a straight line. For a given aesthetic curve we assume that the arc length of the curve is given by s and the radius of curvature ρ, the horizontal axis of the LDDC measures $\log \rho$ and the vertivcal axis $\log(ds/d(\log \rho)) = \log(\rho\, ds/d\rho)$. Since its LDDC is given by a straight line, there exists some constant α and the folloing equation is satisfied:

$$\log(\rho \frac{ds}{d\rho}) = \alpha \log \rho + C \qquad (1)$$

where C is a constant. We call this the fundamental equation of aesthetic curves. Rewrite Eq.(1) and it becomes

$$\frac{1}{\rho^{\alpha-1}} \frac{ds}{d\rho} = e^C = C_0 \qquad (2)$$

Hence there is some constant c_0 such that

$$\rho^{\alpha-1} \frac{d\rho}{ds} = c_0 \qquad (3)$$

From the above equation, if $\alpha \neq 0$, we obtain the first general equation of the aesthetic curve:

$$\rho^\alpha = c_0 s + c_1 \qquad (4)$$

and if $\alpha = 0$, we do the second general equation of the aesthetic curve:[2]

$$\rho = c_0 e^{c_1 s}. \qquad (5)$$

2.2 Input of log-aesthetic curves

Yosida and Saito proposed a method to input an aesthetic curve segment by specifying its two end points and two tangent directions there with three so-called control points for a given α value[4]. However their method can generate only one segment of monotonically increasing or decreasing curvature and can not deal with complex curves with partially increasing and decreasing curvature or those whose segments have various α values. It is difficult to generate complicated curves used for the industrial design in practice. Hence in this paper we propose a method to generate not only a monotonic-rhythm curve but also compounded-rhythm curve.

3 Input Algorithm

3.1 Outline of the algorithm

Log-aesthetic curves including the compounded-rythm curve are generated by the following steps:

1. Input a sequence of points by use of a liquid-crystal pen tablet.

2. Generate a B-spline curve from the sequence of the points to satisfy a given threshold value of approximation.

3. Subdivide the B-spline curve at the points of the extremal value of curvature.

4. Generate several log-aesthetic curves for each segment of the B-spline curve with several given values of α and select the one with the highest degree of approximation.

3.2 Input of point sequence with liquid-crystal pen tablet

We use a liquid-crystal pen tablet to input curves. The designers can directly input a curve as a sequence of points and simultaniously display the curve on the tablet. Hence we can have the same feeling to design curves with a pen and a piece of paper when we input them to the computer. Figure 2 shows how to input curves.

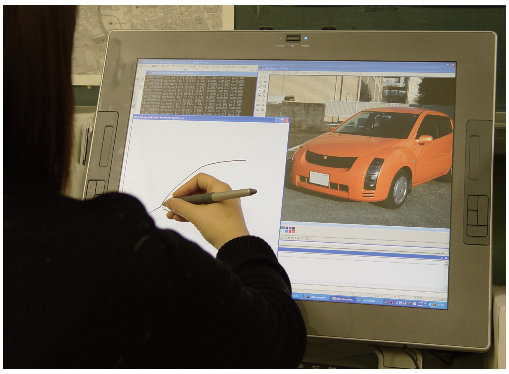


Figure 2: Input of curves with liquid-crystal pen tablet

3.3 B-spline approximation of point sequence

We approximate the sequence of points by a B-pline curve that is one of the standard parametric curves in industry by the least-square method. We adopt a curve of degree three and the objective function for the least-square method is the sum of the squares of the distances between the sequence of the points and the corresponding points on the B-spline curve. We start with one segmented B-spline curve and if the degree of the approximation is not good enough compared with a given threshold value, we increase the number of the segments one by one to satisfy the threshold value. Figure 3 shows an example of approximation results. The black points are input, the pink line is the B-spline curve, and the green crosses are the contorl points of the B-spline curve.

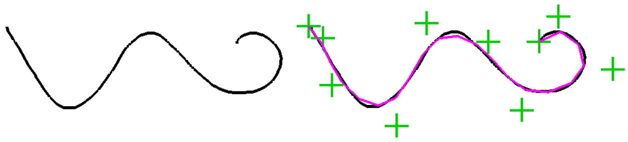


Figure 3: B-Spline curve approximation

3.4 Segmentation of B-spline curve with curvature

In order to apply the method developed by Yoshida and Saito[4], we suvdivide the B-spline curve at the inflections and the curvature extrema. As each segment of a cubic B-spline curve is equivalent to a cubic Bézier curve, we explain how to calculate the inflections and curvature extrema of the cubic Bézier curve instead of the B-spline curve in the following discussion.

3.5 Inflection point

We assume that a cubic Bézie curve is given by $\boldsymbol{C}(t) = (x(t), y(t))$, $0 \leq t \leq 1$ and the 1st and 2nd derivatives of $x(t)$ and $y(t)$ with respect to t is expressed by $\dot{x}(t)$, $\dot{y}(t)$, $\ddot{x}(t)$, and $\ddot{y}(t)$, respectively. We use $f(t)$ and $g(t)$ defined by $f(t) = \dot{x}(t)^2 + \dot{y}(t)^2$, $g(t) = \dot{x}(t)\ddot{y}(t) - \ddot{x}(t)\dot{y}(t)$. Then the curvature with positive or negative sign $\kappa(t)$ is giveb by

$$\kappa(t) = \frac{g(t)}{f(t)^{\frac{3}{2}}} \quad (6)$$

If the curve is not degenerated, we can assume $f(t) \neq 0$ and the parameter values which give points where the curvature is equal to 0 are obtained by solving $g(t) = 0$. $g(t)$ is cubic polynomial of the parameter t and it can be solved analytically.

3.6 Extremum

The extrema are obtained by solving $d\kappa(t)/dt = 0$:

$$\begin{aligned} \frac{d}{dt}\kappa(t)^2 &= 2\kappa(t)\frac{d}{dt}\kappa(t) \\ &= \frac{d}{dt}\frac{g(t)^2}{f(t)^3} \\ &= \frac{g(t)(2\dot{g}(t)f(t) - 3g(t)\dot{f}(t))}{f(t)^4} = 0 \end{aligned}$$

Since the points where $\kappa(t) = 0$ are classified as inflection, we solve the following equation:

$$h(t) = 2\dot{g}(t)f(t) - 3g(t)\dot{f}(t) = 0 \qquad (7)$$

Because $f(t)$ is of degree 4, $h(t)$ is of degree 6. We use some numerical method to get the solutions. For example, we subdivide the interval from 0 to 1 with an equal size and for each subdivided interval, if the signs of $h(t)$ at the two ends are different, the initial value is set to be the middle point of the subdivided interval and the accuracy of the solution can be increased by Newton's method.

Figure 4 shows the control points of the cubic Bézier curve converted from the segments of the B-spline curve. In the figure, the + signs indicate the control points of the Bézier curve and the green lines are generated by connecting the control points. Figure 5 shows cubic Bézier curves subdivided at the inflections and curvature extrema.

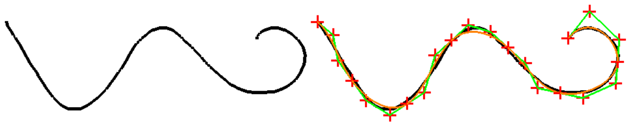


Figure 4: Conversion to a set of cubic Bézier curves

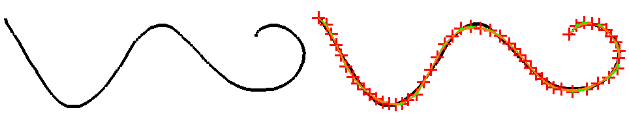


Figure 5: Subdivision at inflection and extrema of curvature

3.7 Generation of log-aesthetic curve

Each of the cubic Bézier curves obtained by subdividing at the inflections and the curvature extrema is approximated by a log-aesthetic curve. In order to apply Yoshida and Saito's method[4], we calculate the intersection point between two lines who have one of the two end points of the curve and whose directions are equal to the tangent vectors there. We specify α: the slope of the logarithmic curvature graph and generate a log-aesthetic curve by use of the two end points and the intersection point. Figure 6 shows generated log-aesthetic curves: the curves in blue color have $\alpha = 0.1$ and those in green do $\alpha = -0.1$.

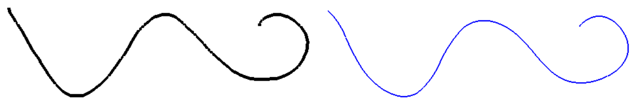


Figure 6: Generated log-aesthetic curve

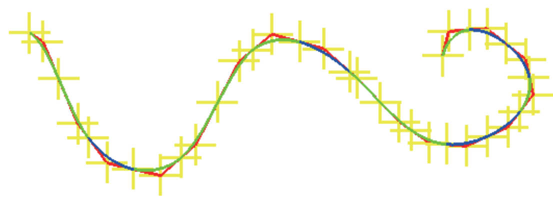


Figure 7: α values of log-aesthetic curves(Blue:0.1, Green:$-$01)

Figure 8 shows curvature of the B-spline and log-aesthetic curves.

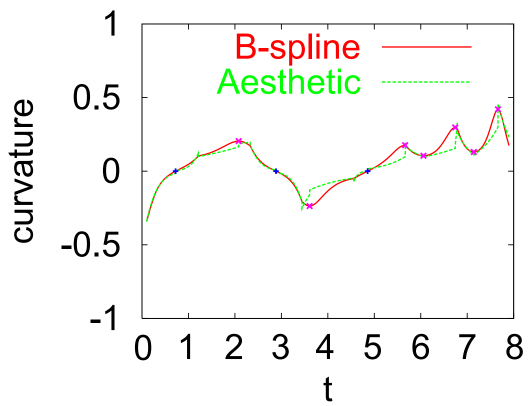


Figure 8: Curvature of B-spline and log-aesthetic curves

Figure 9 shows an example of generated curves simpler than that in Fig.6 to clarify the relationship between the shape and curvature of the generated curves

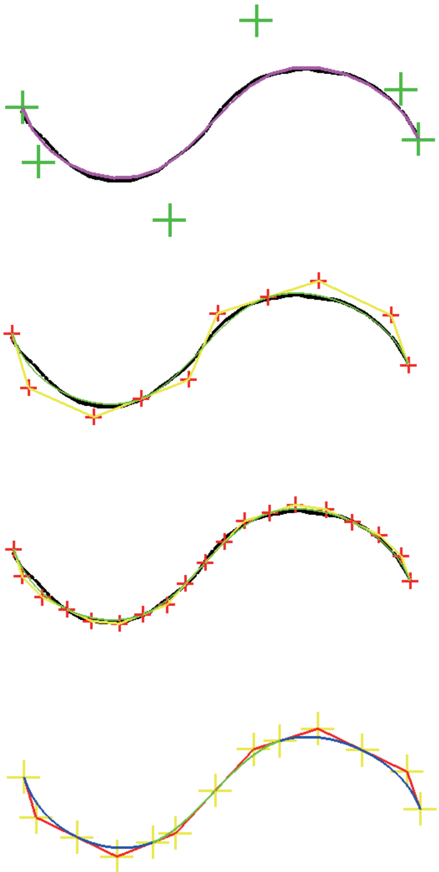


Figure 9: Simple log-aesthetic example

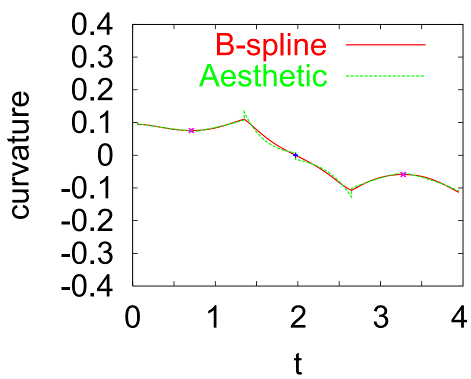


Figure 10: Curvature of B-spline and log-aesthetic curves

The curvature of the B-spline curve changes smoothly as a whole, but the slope of the graph changes gradually and the curve is not sharp and is in a dull tone. In contrast, the change of the curvature of each segment of the log-aesthetic curve is sharp. The difference between the B-spline and log-aesthetic curves on the display is not so clear, but if they are used, for instance, for the character lines of the car bodies, their shapes are critically different.

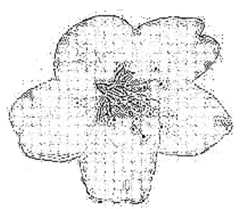
4 Implementation and Discussions

Using log-aesthetic curves generated by our system, we created Fig.11(b) based on Fig.11(a).

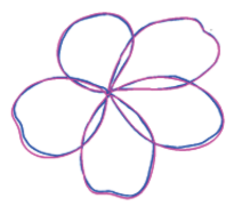
Figures 12 and 13 show several examples of the usage of the log-aesthetic curves to express the outlines of a car body and a musical instrument. For the industrial products, we can reconstruct their outlines by use of the log-aesthetic curves.

(a) Cherry blossom(photo)

(b) Drawing with log-aesthetic curve



(c) Contour extraction of cherry blossom(photo)



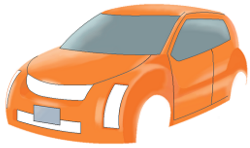
(d) Hand drawing(blue) and log-aesthetic curve(red)

Figure 11: Comparion using a natural object

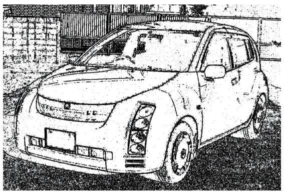
5 Conclusions

In this paper, we have proposed a method to generate log-aesthetic curves which can have several segments with different α vales including the

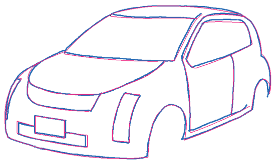
(a) Car(photo)



(b) Drawing with log-aesthetic curve



(c) Contour extraction of car



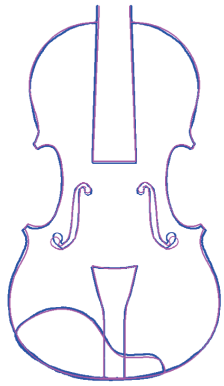
(d) Hand drawing(blue) and log-aesthetic curve(red)

Figure 12: Comparison using an industrial product

(a) Violin(photo)

(b) Drawing with log-aesthetic curve

(c) Contour extraction



(d) Hand drawing(blue) and log-aesthetic curve(red)

Figure 13: Comparison using a musical insturument

compounded-rythm curve. We input a sequence of points as a curve by use of a liquid-crystal pen tablet, approximate it by a B-spline curve, subdivide the B-spline curve at curvature-extremal points, then again approximate each segment of the subdivided B-spline curve by a log-aesthetic curve to generate a curve with complex rhythm.

For future work, we will develop a technique to merge several segments if they can be well approximated by a single log-aesthetic curve segment. We will devise a generation method of the log-aesthetic curve in consideration for the pressure and the inclination, and the speed of the input pen and develop a styling CAD system using the log-aesthetic curve.

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