Adaptive refinement for B-spline subdivision curves

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Abstract

The control polygons of B-spline subdivision curves are usually refined uniformly using a technique called knot-doubling. This uniform refinement approach would perform unnecessary subdivision steps on portions already close to the limit curve enough and, consequently, cause unnecessary (exponential) increase on the number of line segments in the refined polygons. This paper overcomes this problem by proposing a local refinement technique for the control polygons of B-spline subdivision curves. Local refinement is achieved by selectively inserting new knots at midpoints of knot intervals. Efficient adaptive subdivision can be easily realized based on the new technique.

Key words: B-splines, Knot insertion, Subdivision curves

1 Introduction

Subdivision curves and surfaces are powerful tools for graphical modeling and animation because of their scalability, numerical stability, simplicity in coding and, especially, their ability to represent complex shapes of arbitrary topology . They have already been used to represent free-form surfaces in several commercial systems. Doo-Sabin and Catmull-Clark subdivision surfaces are two of the most popular subdivision schemes. These subdivision surfaces are based on uniform tensor product B-spline surfaces, whose non-uniform rational extension (NURBS) is an industry standard in computer graphics as well as CAD/CAM systems.

A subdivision curve is the limit curve of a sequence of line segments generated by iteratively refining a given control polygon. The refining process is usually performed uniformly on all the segments of the current polygon using a technique called *knot-doubling*. This uniform subdivision approach would perform unnecessary subdivision steps on regions that are already flat enough and, consequently, cause unnecessary (exponential) increase on the number of faces in the resulting mesh.

In this paper, a technique to solve the above problem is proposed. The technique performs local refinement of a subdivision curve by selectively inserting knots at midpoints of knot intervals. The new technique is named *SNUS* for *selective non-uniform subdivision* because the selective knot insertion process is similar to that of non-uniform recursive subdivision curves (NURSC) [10], which generalize non-uniform B-spline curves by assigning knot intervals to vertices or edges of the control polygons. Efficient adaptive subdivision can be easily realized based on the new technique as shown in the paper.

The remainder of this paper is organized as follows. Related works in adaptive subdivision for subdivision and parametric surfaces are presented in Section 2. Selective non-uniform subdivision techniques for curves are given in Sections 3 and 4. Concluding remarks and future research directions are given in Section 5.

2 Related work

Subdivision defines smooth surfaces as the limit of a sequence of the refinement of polygonal meshes. For regular patches, this sequence can be defined by knot insertion [2, 3, 9]. The Oslo algorithm is well known for the knot insertion scheme for univariate B-splines [3]. Given a set of knots and control points, a function is constructed by control points with breakpoints at the knots. When a new knot is inserted in a knot sequence, the knot insertion scheme updates neighbor control points for representing the same function.

Early subdivision schemes were designed for generalizing this knot insertion scheme to irregular meshes [5, 1, 7]. Since knot intervals were uniformly determined, knots were not represented explicitly. In this meaning, these schemes can be said special cases of knot insertion, in which knots are inserted uniformly at the midpoints of the knot intervals and the number of knots is doubled during each insertion step.

As a subdivision scheme that allows non-uniform knot intervals, Sederberg et al.[10] proposed non-uniform recursive subdivision surfaces (NURSS). They showed that non-uniform knot intervals could be used for controlling the limit surfaces with creases. However, their scheme was still based on knot-doubling, and non-uniform knot intervals were not discussed as a means of adaptive subdivision.

Adaptive subdivision for irregular meshes is one of the future research trends [12]. We believe that knot insertion is one of the most promising schemes for adaptive subdivision.

Another type of approach to adaptive subdivision is to construct schemes that allow for smooth transitions between uniform meshes of different levels. Zorin et al.[13] maintained control points of each subdivision step using hierarchical structures, and realized multiresolution editing of hierarchical meshes . He also described variations of adaptive subdivision using the similar approach [14]. Kobbelt [6] and Velho et al. [11] subdivide only locally specified portions of a uniform mesh to adaptively refine areas of interest. However these approaches do not have the piecewise functional representations that makes analyzing B-splines easier[12].

3 Curve SNUS

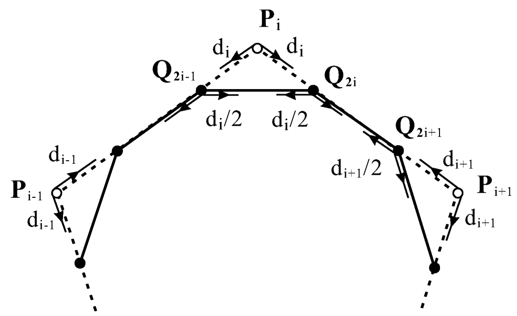
3.1 Curve Knot-Doubling

The control polygon of a periodic B-spline subdivision curve is refined by repetitive *knot-doubling*. *Knot-doubling* here refers to the process of inserting a new knot at the midpoint of each current knot interval[10]. This process doubles the number of control points that represents the same curve. For a non-uniform quadratic periodic B-spline curve, each vertex of the control polygon corresponds to a single quadratic curve segment and a knot interval d_i is assigned to each control vertex $\boldsymbol{P}_i$. A knot-doubling process in this case generates the following new control points $\boldsymbol{Q}_k$:

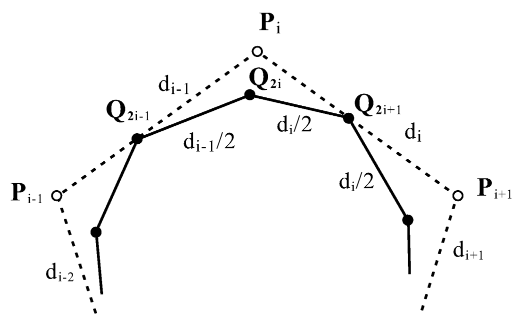
$$\boldsymbol{Q}_{2i} = \frac{(d_i + 2d_{i+1})\boldsymbol{P}_i + d_i\boldsymbol{P}_{i+1}}{2(d_i + d_{i+1})} \tag{1}$$

$$\boldsymbol{Q}_{2i+1} = \frac{d_{i+1}\boldsymbol{P}_i + (2d_i + d_{i+1})\boldsymbol{P}_{i+1}}{2(d_i + d_{i+1})} \tag{2}$$

as shown in Figure 1(a).



(a) Quadratic curve.



(b) Cubic curve.

Figure 1: **Non-uniform B-spline curve**.

For a periodic cubic B-spline curve, each edge of the control polygon corresponds to a single cubic curve segment and the knot intervals are assigned to its edges instead of its vertices. New control points $\boldsymbol{Q}_k$ in this case are calculated by

$$\boldsymbol{Q}_{2i+1} = \frac{(d_i + 2d_{i+1})\boldsymbol{P}_i + (d_i + 2d_{i-1})\boldsymbol{P}_{i+1}}{2(d_{i-1} + d_i + d_{i+1})} \quad (3)$$

$$\boldsymbol{Q}_{2i} = \frac{d_i\boldsymbol{Q}_{2i-1} + (d_{i-1} + d_i)\boldsymbol{P}_i + d_{i-1}\boldsymbol{Q}_{2i+1}}{2(d_{i-1} + d_i)} \quad (4)$$

as illustrated in Figure 1(b).

The above non-selective subdivision scheme by knot-doubling is problematic where the knot intervals are equal to 0 or much smaller than the other intervals. For example, if one of the knot intervals d_i of a quadratic B-spline curve is equal to 0, Equations (1) and (2) give us $\boldsymbol{Q}_{2i-1} = \boldsymbol{Q}_{2i} = \boldsymbol{P}_i$, i.e., the two consecutive control vertices are identical. This means that the number of control vertices would increase but the curve would not be refined. This is the result of inserting a knot into the joint of two adjacent segments. Further knot-doubling processes there would actually slow down the convergence to its limit curve because it would only accumulate control vertices at the same location. Similar problem would also occur around vertices whose knot intervals are much smaller than the others.

For a cubic curve, the similar accumulation of control vertices is unavoidable on an edge whose knot interval d_i is equal to 0 or much smaller than the others. Equations (3) and (4) are simplified as follows when $d_i = 0$:

$$\boldsymbol{Q}_{2i+1} = \frac{d_{i+1}\boldsymbol{P}_i + d_{i-1}\boldsymbol{P}_{i+1}}{(d_{i-1} + d_{i+1})} \quad (5)$$

$$\boldsymbol{Q}_{2i} = \frac{\boldsymbol{P}_i + \boldsymbol{Q}_{2i+1}}{2} \quad (6)$$

The above equations tell us that $\boldsymbol{Q}_{2i-1}$, $\boldsymbol{Q}_{2i}$, and $\boldsymbol{Q}_{2i+1}$ are on the same edge and the middle point $\boldsymbol{Q}_{2i}$ does not contribute to the refinement.

3.2 Selective Knot Insertion

A simple solution to the above problem is not to insert a knot into the joint of two consecutive segments or into a small knot interval segment selectively. For a quadratic curve, you should not 'cut a corner' to stop inserting a knot and it is straightforward to select effective knot insertions if you have appropriate criteria. The cubic curve case is slightly more complicated, but still straightforward enough as is explained below.

As shown in Figure 2, a knot insertion at the midpoint of the initial knot interval d_i of the non-uniform cubic B-spline curve can be achieved by the following update equations (cf. [9]):

$$\boldsymbol{R}_{i-1} = \frac{d_{i+1}\boldsymbol{P}_{i-1} + \{2(d_{i-1} + d_i) + d_{i+1}\}\boldsymbol{P}_i}{2(d_{i-2} + d_{i-1} + d_i)} \quad (7)$$

$$\boldsymbol{R}_i = \frac{(d_i + 2d_{i+1})\boldsymbol{P}_i + (d_i + 2d_{i-1})\boldsymbol{P}_{i+1}}{2(d_{i-1} + d_i + d_{i+1})} \quad (8)$$

$$\boldsymbol{R}_{i+1} = \frac{\{d_i + 2(d_{i+1} + d_{i+2})\}\boldsymbol{P}_{i+1} + d_i\boldsymbol{P}_{i+2}}{2(d_i + d_{i+1} + d_{i+2})} \quad (9)$$

A midpoint insertion for the next initial knot interval d_{i+1} generates three new control vertices $\boldsymbol{S}_i$, $\boldsymbol{S}_{i+1}$, and $\boldsymbol{S}_{i+2}$. Simple algebra shows that they are given by the same equations as Equations (4), (3), and (9)[1], respectively. Further insertions can be performed by applying the updating process of the control vertices described by these equations.

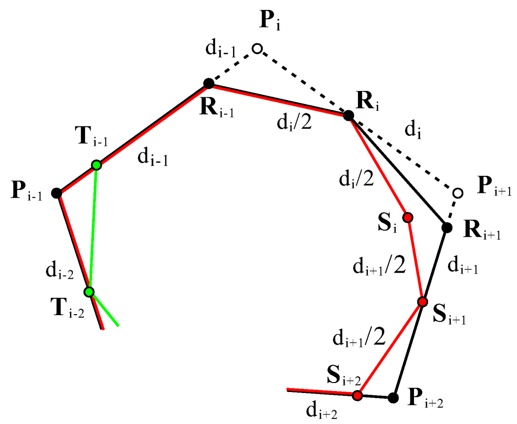


Figure 2: **Selective knot insertion.**

Note that Equations (7) and (9) are similar to Equations (1) and (2) in the sense that the new vertices $\boldsymbol{R}_{i-1}$ and $\boldsymbol{Q}_{2i}$ are moved from the original vertices $\boldsymbol{P}_i$ and $\boldsymbol{P}_{i+1}$ to new locations on edge $\boldsymbol{P}_i\boldsymbol{P}_{i+1}$. Even though we perform another midpoint insertion for the previous knot interval to the unsubdivided knot interval in the other direction, shown in Figure 2 as green line segments, the location of $\boldsymbol{R}_{i-1}$ remains the same. We will use these facts later in the selective subdivision for Catmull-Clark surfaces.

[1] Equation (9) should be applied for the segment $\boldsymbol{P}_i\boldsymbol{P}_{i+1}$ instead of $\boldsymbol{P}_{i+1}\boldsymbol{P}_{i+2}$.

4 SNUS in parameter space

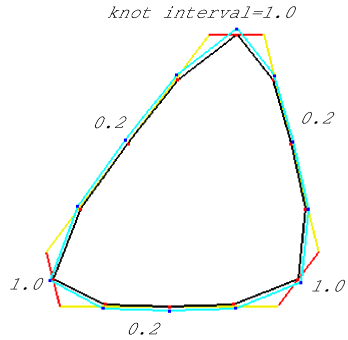
We can apply our technique naturally to NURS curves in the parameter space and use the values of the knot intervals as a criterion for selecting knot insertion locations. One of the typical examples which definitely need SNUS is degree-elevated subdivision curves. The converted curve is represented with multiple knots and many of its knot intervals are equal to 0. The SNUS can avoid inserting knots to the polygon where the knot intervals are equal to 0 or relatively very small.

Figure 3 shows cubic B-spline curve examples subdivided with the SNUS in the parameter space. Initially, a three-sided uniform quadratic B-spline subdivision curve is converted to a six-sided nonunifrom cubic B-spline subdivision curve. The initial knot intervals of the yellow and red edges of the cubic B-spline curve are supposed to be 0 and 1, respectively. To clarify the effect of the SNUS, 0.2 is assigned to the yellow edges instead of 0 and the control polygon for the SNUS to be applied are scaled up a little. In each figure, the standard subdivision generated polygons shown in black with red control points and the SNUS did cyan polygons with blue control points. The SNUS stoped inserting knots at the knot intervals whose value are less than 0.1. In figure 3(a) and (b), both subdivision methods generate the same control polygons, but in (c) some control points are not generated in the SNUS case because the knot intervals are less than 0.1. We can see the accumulation of the control points in the standard subdivisoncase as shown especially in (d).

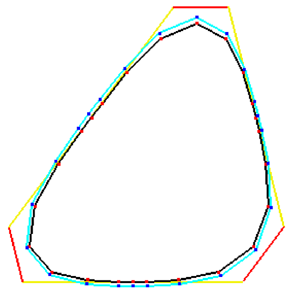
5 Conclusion

This paper presents a new technique, the SNUS, for local refinement of B-spline subdivision curves, which selectively inserts new knots at midpoints of knot intervals. In the proposed scheme, the limit points of the all vertices are guaranteed to be on the limit curve of the original polygon.

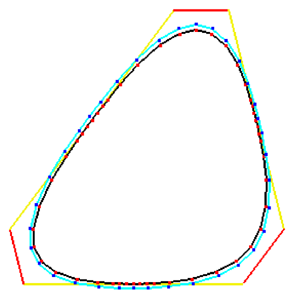
One of the future research topics is on local refinement by inserting knots at arbitrary positions instead of midpoints. Since the quality of adaptive subdivision depends heavily on subdivision criteria, additional work should be devoted to such criteria to extract the maximum power of the SNUS.



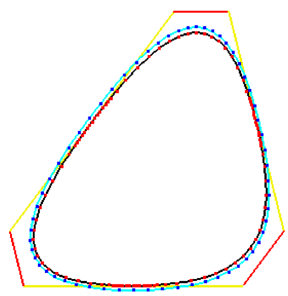
(a) depth=1.



(b) depth=2.



(c) depth=3.



(d) depth=4.

Figure 3: **SNUS in parameter space.**

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